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ISOTROPIC TWO-DIMENSIONAL PSEUDO-RIEMANNIAN METRICS UNIQUELY CONSTRUCTED BY A GIVEN CURVATURE

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Abstract. We prove a global uniqueness theorem of reconstruction of a two-dimensional pseudo-metric by a given Gaussian curvature.

1. Introduction

Local existence and uniqueness theorems of reconstruction of a two-dimensional pseudo-Riemannian isotropic metric

$$ds^2 = 2e^{u(x,y)}dxdy \tag{1.1}$$

in special domains $\Omega \subset \mathbb{R}^2$ based on a given Gaussian curvature

$$K(x,y) = -e^{-u}u_{xy} \tag{1.2}$$

and trace of u on a part of boundary $\partial\Omega$ were proved in [1]. Such metrics play an important role in general relativity and cosmology. Therefore, the problem of global reconstruction of this metric by a given Gaussian curvature seems interesting.
The problem of proving the global existence and uniqueness of this metrics is of special importance. In this paper, we handle the uniqueness part, leaving the question of global existence open. We prove a global uniqueness theorem for domains strictly convex with respect to characteristics of the wave operator $\Box = \partial_x\partial_y$ based on a simple Carleman estimate for this operator.

2. Carleman Estimate for the wave operator $\Box$

Let Ω be an open set in $\mathbb{R}^2$ such that

$$\Omega \subset \{(x,y) \in \mathbb{R}^2 \mid x > 0,\, y > 0\}.$$

We say that Ω is strictly convex with respect to characteristics of the wave operator $\Box$ if for any $(x,y) \in \Omega$ characteristics rays starting at (x,y) and going down and to the left (see fig.3.1) intersect the boundary $\partial\Omega$ just in two points $(x,a(x))$ and $b(y),y)$ where

$$0 \le a(x) < y \quad , \quad 0 \le b(y) < x. \tag{2.1}$$

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