Accepted Manuscript

Prescribed curvature tensor in locally conformally flat manifolds

Romildo Pina, Mauricio Pieterzack

PII: S0393-0440(17)30233-4
DOI: https://doi.org/10.1016/j.geomphys.2017.09.014
Reference: GEOPHY 3078

To appear in: *Journal of Geometry and Physics*

Received date : 29 August 2016
Revised date : 26 September 2017
Accepted date : 28 September 2017

Please cite this article as: R. Pina, M. Pieterzack, Prescribed curvature tensor in locally conformally flat manifolds, *Journal of Geometry and Physics* (2017), https://doi.org/10.1016/j.geomphys.2017.09.014

PRESCRIBED CURVATURE TENSOR IN LOCALLY CONFORMALLY FLAT MANIFOLDS

ROMILDO PINA AND MAURICIO PIETERZACK

Abstract. A global existence theorem for the prescribed curvature tensor problem in locally conformally flat manifolds is proved for a special class of tensors R. Necessary and sufficient conditions for the existence of a metric $\bar{g}$, conformal to Euclidean g, are determined such that $\bar{R} = R$, where $\bar{R}$ is the Riemannian curvature tensor of the metric $\bar{g}$. The solution to this problem is given explicitly for special cases of the tensor R, including the case where the metric $\bar{g}$ is complete on $\mathbb{R}^n$. Similar problems are considered for locally conformally flat manifolds.

1. Introduction

Over the last decades several authors have considered the following problem:

(P) Given a smooth function $\bar{K} : M \to \mathbb{R}$ on a manifold (M, g) is there a metric $\bar{g}$ conformal to g whose scalar curvature is $\bar{K}$?

This problem has been studied by various authors. Particularly, when $\bar{K}$ is a constant it is known as the Yamabe Problem. If $M = \mathbb{R}^n$ with $n \geq 3$ and g is the Euclidean metric, various results can be found in [1], [2], [3] and in their references.

An interesting problem related to problem (P), that is currently under extensive investigation is the prescribed Ricci curvature equation. It can be formulated as follows:

(P1) Given a symmetric $(0,2)$-tensor T, defined on a manifold M^n, $n \geq 3$, does there exist a Riemannian metric g such that $Ric\, g = T$?

When T is nonsingular, that is, its determinant does not vanish, a local solution of the Ricci equation always exists, as shown by DeTurck in [4]. When T is singular, the Ricci equation still admits local solutions, provided that T has constant rank and satisfies certain conditions [5]. Rotationally symmetric nonsingular tensors were considered in [6]. Related results can be found in [5], [7], [8], [9], [10], [14], [11], [12], and the references therein. Recent developments on problem (P1) can be found in [15], [16], [17], and [18].

Another problem related to problem (P1) is the *Prescribed Curvature Tensor problem*, which can be formulated as follows:

(P2) Given a $(0,4)$-tensor R, defined on a manifold M^n, $n \geq 3$, does there exist a Riemannian metric g such that $R_g = R$, where R_g is the Riemannian curvature tensor of the metric g?

Date: August, 2016 and, in revised form, .

2010 *Mathematics Subject Classification.* Primary 53C21; Secondary 35N10.

Key words and phrases. Conformal metric, Riemannian curvature tensor, scalar curvature, Ricci curvature.

The authors were supported in part by CAPES/PROCAD and FAPEG/GO.