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ON GRADIENT QUASI-EINSTEIN SOLITONS

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Abstract. In this paper we consider gradient quasi-Einstein solitons. We establish several formulas, based on which we can get rigid properties for these compact solitons. We also prove that any complete locally conformally flat gradient quasi-Einstein soliton of dimension $n \geq 3$ is locally a warped product with $(n-1)-$dimensional fibers of constant curvature, around any regular point of the potential function.

1. Introduction

Let M^n be a complete Riemannian manifold. We say that (M, g) is a gradient Ricci soliton if there exists a smooth potential function f on M such that the metric g satisfies

$$\mathrm{Ric} + \mathrm{Hess} f = \lambda \mathrm{g}$$

for some soliton constant $\lambda \in \mathbf{R}$. Gradient Ricci solitons arise as limits of dilations of singularities of the Ricci flow. We can refer to [2, 14, 15] and the references therein for the research of gradient Ricci solitons.

We call a metric g τ-quasi-Einstein with potential function f , if for some constant λ,

$$\mathrm{Ric} + \mathrm{Hess} f = \frac{\mathrm{d}f \otimes \mathrm{d}f}{\tau} + \lambda \mathrm{g},$$

where $\tau \neq 0$. For positive integer τ, the $\tau-$quasi-Einstein metrics are closely relative to the existence of warped product Einstein manifolds [1]. An ∞-quasi-Einstein metric indicates a gradient Ricci soliton metric. We can refer to [4, 5, 18, 19, 20, 21, 22] and the references therein for the research of quasi-Einstein metrics.

Gradient Ricci solitons were generalized along different directions. we can refer to [5, 8, 9, 10, 16, 23] and the references therein. In particular, the authors of [9, 10] introduced the gradient Einstein soliton, which is a natural extension of the gradient Ricci soliton. A gradient Einstein soliton (M, g) is defined by

$$\mathrm{Ric} + \mathrm{Hess} f = \rho R \mathrm{g} + \lambda \mathrm{g}$$

for a given potential function f and some constants $\lambda, \rho \in \mathbf{R}$.

In this paper we will study the gradient quasi-Einstein soliton. Let $\rho \neq 0, \tau \neq 0$ be two given constants. We say that (M^n, g) is a gradient $(\rho, \tau)-$quasi-Einstein

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