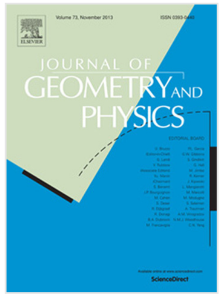
Accepted Manuscript



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Georg Hein, David Ploog

PII: S0393-0440(16)30316-3
DOI: http://dx.doi.org/10.1016/j.geomphys.2016.12.004
Reference: GEOPHY 2899

To appear in: *Journal of Geometry and Physics*

Received date: 1 August 2016
Revised date: 2 December 2016
Accepted date: 8 December 2016

STABILITY OF PICARD SHEAVES FOR VECTOR BUNDLES ON CURVES

GEORG HEIN AND DAVID PLOOG

ABSTRACT. For a stable vector bundle E of slope $\mu(E) > 2g-1$ on a smooth, projective curve of genus g, we show that the Picard sheaf $\hat{E}$ on the Picard variety of the curve is stable. We introduce a homological tool for testing semistability of Picard sheaves. We also obtain the semistability of the general Picard sheaf if $\mu(E) \in [g-2, g], \mu(E) \neq g-1$.

INTRODUCTION

Throughout, X is a smooth, projective genus g curve over an algebraically closed field k. Let $\mathsf{Pic} := \mathsf{Pic}^0(X)$ be the Picard variety of X and $\mathcal{P}$ the Poincaré line bundle on $X \times \mathsf{Pic}$.

For a vector bundle $E \in \mathsf{Coh}(X)$, its *Picard complex* is the Fourier–Mukai (or integral) transform $\hat{E} := \mathsf{FM}_{\mathcal{P}}(E)$, an object of $\mathsf{D}^{\mathrm{b}}(\mathsf{Pic})$. We denote its two cohomology sheaves by $\hat{E}^0$ and $\hat{E}^1$ and call these the *Picard sheaves* of E. Our goal is to show that $\hat{E}$ is (semi)stable on Pic for general, (semi)stable bundles E on X for certain slopes. In fact, we prove this by showing that $\hat{E}$ is semistable when restricted to curves $i\colon X \hookrightarrow \mathsf{Pic}$. We prove the bulk of the following result in Corollaries 2.3 ($\mu > 2g-1$) and 3.4 ($\mu < -1$); the border cases are dealt with in Corollaries 1.13 ($\mu = -1$) and 3.1 ($\mu = 2g-1$):

Theorem A. *If E is a stable bundle on X of slope $\mu(E) > 2g-1$, then the Picard sheaf $\hat{E}^0$ is stable on Pic. Dually, if E is stable of slope $\mu(E) < -1$, then the Picard sheaf $\hat{E}^1$ is stable. The analogous statements hold for semistability, using the non-strict inequalities.*

For this, we employ a new notion of orthogonality of bundles on X; see Definition 1.2. We also use that concept to obtain results about Picard sheaves for generic semistable bundles of slope $\mu \in [g-2, g]$, $\mu \neq g-1$; see Proposition 3.7 and Corollary 3.8.

Theorem B. *For any rational $\mu \in (g-1, g]$, there exists a semistable bundle E on X of slope μ such that its Picard sheaf $\hat{E}^0$ is semistable. Dually, for $\mu \in [g-2, g-1)$, there exists a semistable bundle E on X of slope μ such that its Picard sheaf $\hat{E}^1$ is semistable.*

In order to show Theorem A, we generalise Clifford's theorem about estimating global sections, from divisors to not necessarily semistable vector bundles. If $E = L_1 \oplus \cdots \oplus L_r$ is a direct sum of line bundles with all $\deg(L_i) \in [0, 2g-2]$, then $h^0(L_i) - 1 \leq \deg(L_i)/2$ by the classical Clifford theorem. This sums up to $h^0(E) - r \leq \deg(E)/2$. Therefore, the best generalisation one can hope for is the following result; see Proposition 4.1, where we also give precise information about the equality case.

Proposition C. *Let E be a vector bundle of rank r and degree d on the smooth projective curve X of genus g. If $\mu_{\max}(E) \leq 2g-2$ and $\mu_{\min}(E) \geq 0$, then $h^0(E) - r \leq d/2$.*

The special case of semistable vector bundles of slope $\mu \in [0, 2g-2]$ was already proved in [2, Theorem 2.1]. If the slope of a semistable bundle E is not in this interval, then either $H^0(E) = 0$ or $H^1(E) = 0$, and the dimension of the remaining cohomology group is computed by the Riemann–Roch theorem.

MSC 2010: 14H60; 14F05
Date: November 20, 2015.