International Journal of Engineering Science 46 (2008) 1203–1217

Contents lists available at ScienceDirect

International Journal of Engineering Science

journal homepage: www.elsevier.com/locate/ijengsci

Dispersion in unsteady Couette–Poiseuille flows

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ARTICLE INFO

Article history:
Received 19 May 2007
Accepted 5 June 2008
Available online 24 July 2008

Keywords:
Dispersion coefficient
Periodic pressure gradient
Plate pulsation
Couette–Poiseuille flow
Finite difference
Phase difference

ABSTRACT

The paper presents the longitudinal dispersion of passive contaminant released in an incompressible viscous fluid flowing between two infinite parallel flat walls, in which the flow is driven by the application of both periodic pressure gradient and the oscillation of upper plate in its own plane with a constant velocity. A finite difference implicit scheme has been adopted to solve the unsteady convection-diffusion equation for all time period based on Aris method of moments. The dispersion coefficients are obtained for three different flow situations: steady, periodic and the combined effect of steady and periodic Couette–Poiseuille flows, separately. The results show that oscillation of upper plate produces more dispersion than the pulsation of pressure gradient and their combined action leads to a further increase of dispersion. Also plate oscillation has stronger effect on velocity distribution and on dispersion coefficient than the pressure pulsation. There is a remarkable difference in the behaviour of dispersion coefficient depending on whether the ratio of two frequencies arising from the oscillations of pressure gradient and the upper plate possesses a proper fraction or not.

1. Introduction

The dispersion of a solute of soluble matter injected into a pipe has been extensively studied by many researchers following the classical work of Taylor [1]. Aris [2] extended Taylor's theory including the longitudinal diffusion and developed an approach 'method of moments' to analyze the convection-diffusion process in steady flow using first few integral moments. A complete moment solution valid for all times has been presented by Barton [3] who resolved certain technical difficulties that occurred in Aris method of solution.

Aris [4] used his method of moments to analyze the longitudinal dispersion coefficient of solute in an oscillatory flow of a viscous incompressible fluid within an infinite tube under a periodic pressure gradient. Some important characteristics of the dispersion phenomenon in time-dependent flows within a conduit may be found in Chatwin [5], Smith [6], Jimenez and Sullivan [7], Yasuda [8]. Purtell [9] analysed the effect of flow oscillation (without time mean velocity) due to the periodic pressure gradient on the axial diffusion of solute in a pipe, considering a small perturbation to the oscillation-Reynolds number $Re = \frac{\omega R^2}{\nu}$, where ω is the frequency of oscillation, R is the radius of the tube, and ν is the kinematic viscosity of the fluid. His attention was restricted to the initial distribution of solute as a step function. Mukherjee and Mazumder [10] extended Aris–Barton method of moments for all time evolution of the second central moment of dispersion of passive contaminant in an oscillatory flow within a conduit of uniform cross-section under a periodic pressure gradient. Using generalised dispersion model proposed by Gill and Sankarasubramanian [11], Hazra et al. [12] studied the dispersion of a solute in a oscillatory flow through a channel. Bandyopadhyay and Mazumder [13] extended the analysis of Mukherjee and Mazumder to study the mean, variance and longitudinal dispersion coefficient of tracer material released in an incompressible viscous fluid flowing

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doi:10.1016/j.ijengsci.2008.06.006

through a channel. Erdogan [14] analysed the effect of gravity that arises out due to the density difference between a solute and a solvent on longitudinal dispersion in generalised Couette flow. Bandyopadhyay and Mazumder [15] explained the dispersion phenomena in a liquid flowing between two infinite parallel plates, when the flow is driven by the oscillation of the upper plate in its own plane with a constant velocity as well as by an imposed constant pressure gradient. However, no attempt has been made to evaluate the longitudinal dispersion coefficient of a solute in fluid flow taking into consideration of simultaneous influence of both periodic pressure gradient and the upper plate oscillation with their non-zero means.

The purpose of the present paper is to elucidate the dispersion of tracer particles injected in a fluid flowing through a parallel plate channel under the combined action of oscillation of the upper plate and periodic pressure gradient. The main idea is to replace the constant pressure gradient by time-dependent one in addition to the upper plate oscillation, and then investigate the combined effects of both pressure and plate pulsations on the dispersion process. Owing to the complexity of the analytical solutions of integral moment equations, it has been solved numerically by adopting a finite difference scheme based on Crank–Nicholson implicit method. Results are shown to explain the individual as well as due to the simultaneous action of both driving forces arising from the oscillation of the upper plate and pulsation of pressure gradient on dispersion process.

The study of dispersion under periodic pressure gradient has important applications to the dispersion of tracers in pulsatile blood flow in a cardiovascular system and the discharge of outfalls in homogeneous tidal estuaries. Knowledge of dispersion under oscillation of boundary is important in the hydrodynamic theory of lubrication. This study also has importance to model shear-driven flows encountered in micro motors, micro channels and other micro fluidic systems.

2. Mathematical formulations

Consider an unsteady two-dimensional laminar flow of an homogeneous, incompressible viscous fluid flowing between two infinite parallel plates separated by a distance $2l$ apart. We have used a cartesian coordinate system with x^*-axis along the flow and y^*-axis perpendicular to the flow. The plates are situated at $y^* = \pm l$. Here the flow is assumed to be fully developed and unidirectional along x^*-direction and it is a function of y^* and t^* only. The flow is driven by the combined effect of both axial periodic pressure gradient and the upper plate oscillations. Under these assumptions, the axial velocity component $u^*(y^*,t^*)$ satisfies the Navier–Stokes equation

$$\frac{\partial u^*}{\partial t^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} + \nu\left(\frac{\partial^2 u^*}{\partial y^{*2}}\right) \tag{1}$$

where ρ is the density of the fluid, p^* is the fluid pressure, t^* is the time and ν is the kinematic viscosity of the fluid.

The axial periodic pressure gradient with a non-zero mean P_{x^*} is given by

$$-\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} = P_{x^*}[1 + \epsilon_{\mathrm{p}}\ Re(\mathrm{e}^{\mathrm{i}\omega_{\mathrm{p}} t^*})] \tag{2}$$

and the pulsation of the upper plate at $y^* = l$ with a constant velocity U in its own plane given by

$$u^* = U[1 + \epsilon_w\ Re(\mathrm{e}^{\mathrm{i}\omega_{\mathrm{w}} t^*})] \tag{3}$$

where $\epsilon_{\mathrm{p}} P_{x^*}$ and ω_{p} are respectively the amplitude and frequency of the pressure pulsation; $\epsilon_{\mathrm{w}} U$ and ω_{w} are the amplitude and the angular frequency of the upper plate oscillation respectively. *Re* denotes the real part and subscripts p and w represent respectively the pressure and the wall.

When a passive solute of constant molecular diffusivity D is released in the above mentioned flow through a channel, the concentration $C(x,y,t)$ of the diffusing substance satisfies the non-dimensional convection-diffusion equation as

$$\frac{\partial C}{\partial t} + Pe\ u(y,t)\frac{\partial C}{\partial x} = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)C \tag{4}$$

where the dimensionless quantities are given by

$$x = \frac{x^*}{l},\ y = \frac{y^*}{l},\ t = \frac{Dt^*}{l^2},\ u = \frac{u^*}{U},\ Pe = \frac{Ul}{D} \tag{5}$$

Here the velocity $u(y,t)$ is the combination of steady and periodic flows, D is the molecular diffusivity of the solute and Pe, the Péclet number of the flow, gives the relative magnitude of typical convection speeds compared to typical cross-sectional diffusive speeds. It is also assumed that injected solute does not effect the flow of the carrying fluid.

The initial and boundary conditions to be satisfied by the contaminant input are

$$\left.\begin{array}{l} C(x,y,0) = \mathscr{C}(x,y), \quad y \in [-1,1] \\ \frac{\partial C}{\partial y} = 0 \quad \text{at} \quad y = \pm 1, \\ C \quad \text{finite at all points}, \\ \frac{1}{2}\int_{-1}^{+1}\int_{-\infty}^{+\infty} C(0,x,y)\mathrm{d}x\,\mathrm{d}y = 1, \end{array}\right\} \tag{6}$$