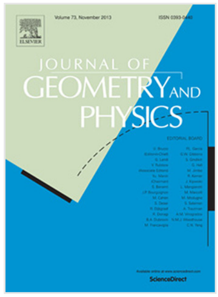


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Helena Albuquerque, Elisabete Barreiro, Antonio J. Calderón Martín, José M. Sánchez Delgado

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# THE STRUCTURE OF LEIBNIZ SUPERALGEBRAS ADMITTING A MULTIPLICATIVE BASIS

HELENA ALBUQUERQUE, ELISABETE BARREIRO, ANTONIO J. CALDERÓN MARTÍN, AND JOSÉ M. SÁNCHEZ DELGADO

ABSTRACT. In the literature, most of the descriptions of different classes of Leibniz superalgebras $(\mathfrak{L} = \mathfrak{L}_{\bar{0}} \oplus \mathfrak{L}_{\bar{1}}, [\cdot,\cdot])$ have been made by given the multiplication table on the elements of a graded basis $\mathcal{B} = \{v_k\}_{k\in K}$ of $\mathfrak{L}$, in such a way that for any $i, j \in K$ we have $[v_i, v_j] = \lambda_{i,j}[v_j, v_i] \in \mathbb{F}v_k$ for some $k \in K$, where $\mathbb{F}$ denotes the base field and $\lambda_{i,j} \in \mathbb{F}$. In order to give a unifying viewpoint of all these classes of algebras we introduce the category of Leibniz superalgebras admitting a multiplicative basis and study its structure. We show that if a Leibniz superalgebra $\mathfrak{L} = \mathfrak{L}_{\bar{0}} \oplus \mathfrak{L}_{\bar{1}}$ admits a multiplicative basis then it is the direct sum $\mathfrak{L} = \bigoplus_\alpha \mathcal{I}_\alpha$ with any $\mathcal{I}_\alpha = \mathcal{I}_{\alpha,\bar{0}} \oplus \mathcal{I}_{\alpha,\bar{1}}$ a well described ideal of $\mathfrak{L}$ admitting a multiplicative basis inherited from $\mathcal{B}$. Also the $\mathcal{B}$-simplicity of $\mathfrak{L}$ is characterized in terms of $J$-connections.

*Keywords*: Leibniz superalgebra, multiplicative basis, infinite dimension, structure theory.
*MSC2010*: 17A32, 17A70, 17A60.

## 1. INTRODUCTION AND PREVIOUS DEFINITIONS

Leibniz superalgebras appear as an extension of Leibniz algebras (see [4, 5, 10, 13, 14, 15, 16, 17]), in a similar way than Lie superalgebras generalize Lie algebras, motivated in part for its applications in Physics. The present paper is devoted to the study of the structure of Leibniz superalgebras $\mathfrak{L}$ admitting a multiplicative basis over a field $\mathbb{F}$. Since a Leibniz algebra is a particular case of a Leibniz superalgebra (with $\mathfrak{L}_{\bar{1}} = \{0\}$), this work extends the results exhibited in [6]. We would like to remark that the techniques used in this paper also hold in the infinite-dimensional case over arbitrary fields, being adequate enough to provide us a second Wedderburn-type theorem in this general framework (Theorems 2.1 and 3.1). Moreover, although we make use of the ideal $\mathfrak{I}$ which is deeply inherent to Leibniz theory, we believe that our approach can be useful for the knowledge of the structure of wider classes of algebras.

**Definition 1.1.** A *Leibniz superalgebra* $\mathfrak{L}$ is a $\mathbb{Z}_2$-graded algebra $\mathfrak{L} = \mathfrak{L}_{\bar{0}} \oplus \mathfrak{L}_{\bar{1}}$ over an arbitrary base field $\mathbb{F}$, with its bilinear product denoted by $[\cdot,\cdot]$, whose homogenous elements $x \in \mathfrak{L}_{\bar{i}}, y \in \mathfrak{L}_{\bar{j}}, \bar{i}, \bar{j} \in \mathbb{Z}_2$, satisfy

$$[x, y] \in \mathfrak{L}_{\bar{i}+\bar{j}}$$

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