

Accepted Manuscript

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PII: S0393-0440(16)30177-2

DOI: <http://dx.doi.org/10.1016/j.geomphys.2016.07.011>

Reference: GEOPHY 2802

To appear in: *Journal of Geometry and Physics*

Received date: 23 February 2016

Revised date: 15 July 2016

Accepted date: 19 July 2016



Please cite this article as: H. Albuquerque, E. Barreiro, A.J.C. Martín, J.M.S. Delgado, The structure of Leibniz superalgebras admitting a multiplicative basis, *Journal of Geometry and Physics* (2016), <http://dx.doi.org/10.1016/j.geomphys.2016.07.011>

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THE STRUCTURE OF LEIBNIZ SUPERALGEBRAS ADMITTING A MULTIPLICATIVE BASIS

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ABSTRACT. In the literature, most of the descriptions of different classes of Leibniz superalgebras ($\mathfrak{L} = \mathfrak{L}_{\bar{0}} \oplus \mathfrak{L}_{\bar{1}}, [\cdot, \cdot]$) have been made by given the multiplication table on the elements of a graded basis $\mathcal{B} = \{v_k\}_{k \in K}$ of \mathfrak{L} , in such a way that for any $i, j \in K$ we have $[v_i, v_j] = \lambda_{i,j}[v_j, v_i] \in \mathbb{F}v_k$ for some $k \in K$, where \mathbb{F} denotes the base field and $\lambda_{i,j} \in \mathbb{F}$. In order to give a unifying viewpoint of all these classes of algebras we introduce the category of Leibniz superalgebras admitting a multiplicative basis and study its structure. We show that if a Leibniz superalgebra $\mathfrak{L} = \mathfrak{L}_{\bar{0}} \oplus \mathfrak{L}_{\bar{1}}$ admits a multiplicative basis then it is the direct sum $\mathfrak{L} = \bigoplus_{\alpha} \mathcal{I}_{\alpha}$ with any $\mathcal{I}_{\alpha} = \mathcal{I}_{\alpha, \bar{0}} \oplus \mathcal{I}_{\alpha, \bar{1}}$ a well described ideal of \mathfrak{L} admitting a multiplicative basis inherited from \mathcal{B} . Also the \mathcal{B} -simplicity of \mathfrak{L} is characterized in terms of J -connections.

Keywords: Leibniz superalgebra, multiplicative basis, infinite dimension, structure theory.

MSC2010: 17A32, 17A70, 17A60.

1. INTRODUCTION AND PREVIOUS DEFINITIONS

Leibniz superalgebras appear as an extension of Leibniz algebras (see [4, 5, 10, 13, 14, 15, 16, 17]), in a similar way than Lie superalgebras generalize Lie algebras, motivated in part for its applications in Physics. The present paper is devoted to the study of the structure of Leibniz superalgebras \mathfrak{L} admitting a multiplicative basis over a field \mathbb{F} . Since a Leibniz algebra is a particular case of a Leibniz superalgebra (with $\mathfrak{L}_{\bar{1}} = \{0\}$), this work extends the results exhibited in [6]. We would like to remark that the techniques used in this paper also hold in the infinite-dimensional case over arbitrary fields, being adequate enough to provide us a second Wedderburn-type theorem in this general framework (Theorems 2.1 and 3.1). Moreover, although we make use of the ideal \mathfrak{J} which is deeply inherent to Leibniz theory, we believe that our approach can be useful for the knowledge of the structure of wider classes of algebras.

Definition 1.1. A *Leibniz superalgebra* \mathfrak{L} is a \mathbb{Z}_2 -graded algebra $\mathfrak{L} = \mathfrak{L}_{\bar{0}} \oplus \mathfrak{L}_{\bar{1}}$ over an arbitrary base field \mathbb{F} , with its bilinear product denoted by $[\cdot, \cdot]$, whose homogenous elements $x \in \mathfrak{L}_{\bar{i}}, y \in \mathfrak{L}_{\bar{j}}, \bar{i}, \bar{j} \in \mathbb{Z}_2$, satisfy

$$[x, y] \in \mathfrak{L}_{\bar{i}+\bar{j}}$$

The first, second and the fourth authors acknowledge financial assistance by the Centre for Mathematics of the University of Coimbra – UID/MAT/00324/2013, funded by the Portuguese Government through FCT/MCTES and co-funded by the European Regional Development Fund through the Partnership Agreement PT2020. The third and the fourth author are supported by the PCI of the UCA ‘Teoría de Lie y Teoría de Espacios de Banach’, by the PAI with project numbers FQM298, FQM7156 and by the project of the Spanish Ministerio de Educación y Ciencia MTM2013-41208P. The fourth author acknowledges the Fundação para a Ciência e a Tecnologia for the grant with reference SFRH/BPD/101675/2014.

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