Accepted Manuscript

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PII: S0393-0440(16)30232-7
DOI: http://dx.doi.org/10.1016/j.geomphys.2016.09.011
Reference: GEOPHY 2837

To appear in: *Journal of Geometry and Physics*

Received date: 11 August 2015
Revised date: 15 September 2016
Accepted date: 25 September 2016

Please cite this article as: D. Viennot, Non-abelian higher gauge theory and categorical bundle, *Journal of Geometry and Physics* (2016), http://dx.doi.org/10.1016/j.geomphys.2016.09.011

Non-abelian higher gauge theory and categorical bundle

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Abstract

A gauge theory is associated with a principal bundle endowed with a connection permitting to define horizontal lifts of paths. The horizontal lifts of surfaces cannot be defined into a principal bundle structure. An higher gauge theory is an attempt to generalize the bundle structure in order to describe horizontal lifts of surfaces. A such attempt is particularly difficult for the non-abelian case. Some structures have been proposed to realize this goal (twisted bundle, gerbes with connection, bundle gerbe, 2-bundle). Each of them uses a category in place of the total space manifold of the usual principal bundle structure. Some of them replace also the structure group by a category (more precisely a Lie crossed module viewed as a category). But the base space remains still a simple manifold (possibly viewed as a trivial category with only identity arrows). We propose a new principal categorical bundle structure, with a Lie crossed module as structure groupoid, but with a base space belonging to a bigger class of categories (which includes non-trivial categories), that we called affine 2-spaces. We study the geometric structure of the categorical bundles built on these categories (which are a more complicated structure than the 2-bundles) and the connective structures on these bundles. Finally we treat an example interesting for quantum dynamics which is associated with the Bloch wave operator theory.

1. Introduction

The geometry of the principal bundles plays an important role in theoretical physics. It is the natural framework to model the fundamental interactions between point particles in classical field theory, and is the startpoint for the quantum field theory [1]. Moreover in nonrelativistic quantum physics, the geometric (Berry) phase phenomenon [2] is closely related to this geometry. A principal bundle naturally arises to treat cyclic quantum dynamics [3] or adiabatic quantum dynamics driven by classical parameters [4, 5, 6, 7]. These physical problems are associated with the holonomies or the horizontal lifts of paths drawn on the base manifold of the principal bundle.
The horizontal lifts of surfaces cannot be defined within the framework of the principal bundles. The interest for the horizontal lifts of surfaces arises from the development of the string and brane theories, in which the string and brane gauge theory is associated with holonomies of surfaces [8, 9, 10, 11, 12]. Recently, we have shown that the geometric phases associated with quantum systems submitted to some decoherence processes take place in higher gauge theories associated with horizontal lifts of surfaces [13, 14, 15].
Geometric realizations of the abelian higher gauge theories are well understood, as gerbes