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Entire solutions originating from monotone fronts to the Allen-Cahn equation

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Abstract

In this paper, we study entire solutions of the Allen-Cahn equation in one-dimensional Euclidean space. This equation is a scalar reaction-diffusion equation with a bistable nonlinearity. It is well-known that this equation admits three different types of traveling fronts connecting two of its three constant states. Under certain conditions on the wave speeds, the existence of entire solutions originating from three and four fronts is shown by constructing some suitable pairs of super-sub-solutions. Moreover, we show that there are no entire solutions originating from more than four fronts.

Keywords: reaction-diffusion equation, traveling front, entire solution, super-sub-solutions
2000 MSC: 35K57, 35K58, 35B08, 35B40.

1. Introduction

In this paper, we consider the following reaction-diffusion equation

$$u_t = u_{xx} + f(u), \quad x \in \mathbb{R},\ t \in \mathbb{R}, \tag{1.1}$$

where the function $f(u) \in C^2(\mathbb{R})$ satisfies

$$f(0) = f(1) = 0, \quad f'(0),\ f'(1) < 0, \tag{1.2}$$

$$f(a) = 0,\ f'(a) > 0,\ a \in (0,1), \quad f(u) \neq 0 \text{ for } u \in (0,a) \cup (a,1), \tag{1.3}$$

$$\int_0^1 f(u)\, du > 0. \tag{1.4}$$

A typical example of f is $f(u) = u(1-u)(u-a)$, where $a \in (0, 1/2)$. This equation is often called the Allen-Cahn equation or the Nagumo equation. It is easy to see that the constant states $u = 0$ and $u = 1$ are stable and the constant state $u = a$ is unstable for the kinetic equation (i.e., (1.1) without diffusion term), since $f'(0) < 0$, $f'(1) < 0$ and $f'(a) > 0$.

Due to the rich dynamics of this prototype equation (1.1), there have been a lot of research on the dynamical behaviors of (1.1). One of the main concerns on the dynamics of (1.1) is the existence of entire solutions. Here an entire solution means a classical solution defined for all $(x,t) \in \mathbb{R}^2$. One of typical examples of entire solutions is the traveling wave solution. A solution u of (1.1) is called a traveling wave solution, if $u(x,t) = \Phi(x + vt)$ for some constant v (the wave speed) and some function Φ (the wave profile). A traveling wave solution is called a *traveling front*, if it connects two different constant states. In fact, (1.1) admits three different kinds of traveling fronts connecting states $\{0,1\}$, $\{0,a\}$, $\{a,1\}$, respectively. The first one is the bistable connection and the latter two cases are the monostable connections. For the reader's

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