Accepted Manuscript

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Grégory Faye

PII: S0167-2789(17)30547-X
DOI: https://doi.org/10.1016/j.physd.2018.04.004
Reference: PHYSD 32020

To appear in: *Physica D*

Received date : 5 October 2017
Revised date : 19 April 2018
Accepted date : 23 April 2018

Traveling fronts for lattice neural field equations

Grégory Faye*[1]

[1]Institut de Mathématiques de Toulouse, UMR 5219, Université de Toulouse CNRS, UPS IMT, F-31062 Toulouse Cedex 9, France

May 5, 2018

Abstract

We show existence and uniqueness of traveling front solutions to a class of neural field equations set on a lattice with infinite range interactions in the regime where the kinetics of each individual neuron is of bistable type. The existence proof relies on a regularization of the traveling wave problem allowing us to use well-known existence results for traveling front solutions of continuous neural field equations. We then show that the traveling front solutions which have nonzero wave speed are unique (up to translation) by constructing appropriate sub and super solutions. The spectral properties of the traveling fronts are also investigated via a careful study of the linear operator around a traveling front in co-moving frame where we crucially use Fredholm properties of nonlocal differential operators previously obtained by the author in an earlier work. For the spectral analysis, we need to impose an extra exponential localization condition on the interactions.

Introduction

For $n \in \mathbb{Z}$, we consider the following lattice differential equation

$$\dot{u}_n(t) = -u_n(t) + \sum_{j\in\mathbb{Z}} K_j S(u_{n-j}(t)), \quad t > 0, \tag{1.1}$$

where $\dot{u}_n$ stands for $\frac{\mathrm{d}u_n}{\mathrm{d}t}$ and $u_n(t)$ represents the membrane potential of neuron labelled n at time t. Here K_j represents the strength of interactions associated to the neural network at position j on the lattice and the firing rate of neurons $S(u)$ is a nonlinear function. Such an equation can be seen as a Hopfield neural network model with infinite range interactions [19] or more simply as a discrete neural field equation [12] where each neuron is set on the lattice $\mathbb{Z}$ with all to all couplings.

*email: gregory.faye@math.univ-toulouse.fr