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Editorial

Introduction to Special Issue: Nonlinear Partial Differential Equations in Mathematical Fluid Dynamics

This special issue on *"Nonlinear Partial Differential Equations in Mathematical Fluid Dynamics"* is dedicated to Professor Edriss S. Titi on the occasion of his sixtieth birthday, which he celebrated in 2017. The original research contributions in this volume highlight the impact that his work has had in advancing the mathematical study of fluid mechanics. Edriss Titi's influence, especially on more than one generation of researchers, is far reaching and the content of the present volume is necessarily a selection of scholarly articles that have been inspired directly or indirectly from his work. This Introduction will similarly highlight only some of his many important results.

Edriss Titi's wide range of research has focused on the development of analytical and computational techniques for investigating nonlinear phenomena. The study of the Euler and the Navier–Stokes equations of incompressible fluid mechanics has a prominent role in his work, but other related nonlinear partial differential equations arising as models in a wide range of applications in nonlinear science and engineering, have been addressed in his extensive record of publications, many of which has been influential and transformative in the field. The applications include, but are not limited to, fluid mechanics, geophysics, turbulence, chemical reactions, nonlinear fiber optics, and control of complex systems.

Edriss Titi received his M.Sc. in Mathematics in 1981 from Technion, Haifa, Israel, and his Ph.D. in Applied Mathematics from Indiana University, Bloomington in 1986, under the supervision of Ciprian Foias, with whom he continued to collaborate for many years. His early work already underscores a broad spectrum of interests, stemming from earlier results of Constantin, Foias, and Temam, combining a dynamical systems approach with analytical techniques to the study the fundamental equations of fluid dynamics.

After a postdoctoral appointment as *L.E. Dickson Instructor* in the Department of Mathematics at the University of Chicago, Edriss Titi joined the Mathematics Department at the University of California, Irvine, in 1988, where he held a joint appointment in the Department of Mathematics and the Department of Mechanical and Aerospace Engineering until he retired in 2013. Since 2003 Edriss Titi has also held a joint appointment at the Weizmann Institute of Science in Israel as Professor of Computer Science and Applied Mathematics. After retiring from UC-Irvine, in 2014 he joined the faculty at Texas A&M University where he is currently the *Arthur Owen Professor of Mathematics.*

One of Edriss Titi's most influential early work is undoubtedly the 1994 work [1] on Onsager's conjecture with Constantin and E, where they established the positive part of Onsager Conjecture, namely, that weak solutions of the Euler equations with more than 1/3 derivative conserve energy. The proof of this result is elegant and surprisingly short. Only decades later Onsager's Conjecture was finally proved in its entirety by Isett [2], exploiting convex integration and the theory of wild solutions developed by De Lellis, Székelyhidi and collaborators [3]. Recently, Bardos, Titi and Wiedemann [4,5] have studied the physically relevant case of energy conservation in bounded domains.

Edriss Titi also served as a consultant for many years at Los Alamos National Laboratory, where he was a frequent visitor. Professor Titi was the *Orson Anderson Distinguished Visiting Scholar* (1997–1998) at the Institute of Geophysics and Planetary Physics and the *Stanislaw M. Ulam Distinguished Visiting Scholar* (2002–2003) at the Center for Nonlinear Studies in the Los Alamos National Laboratory. The time at Los Alamos consolidated a prolific collaboration with Holm and others resulting, over time, in the development of the Leray-alpha [6], Viscous Camassa–Holm [7–11], Euler-Voigt models [12,13] and the study of other physically-based reduced model of turbulence [14,15,12,16,17].

One hallmark of Edriss Titi's work is the use of rigorous quantitative analysis to gain insight on physically motivated problems. With Mahalov and Leibovich [18], he proved that helical flows form an invariant subspace for the Navier–Stokes equations, and later consider helical solutions of the 3D Euler equations with Ettinger [19]. These results, although intuitive and generally accepted in the community, had not been previously justified rigorously. Helical flows are one example of exact solutions of the fluid equations that admit a one-dimensional symmetry group, other important example include axi-symmetric and two-and-a-half dimensional flows. It is a physically relevant and mathematically intriguing question whether these flows are stable under three dimensional perturbations in both the viscous and inviscid case. Two decades later Edriss Titi, in collaboration with Bardos, Lopes Filho, Niu, and Nussenzveig Lopes [20], showed that indeed stability holds in the class of weak solutions for Navier–Stokes flows, in the sense that the symmetry-preserving weak solution is unique. In the inviscid case, wild solutions provide examples of breaking of the symmetry.

Another important aspect of Edriss Titi's research is the attention to numerical methods and their interplay with theoretical studies and modeling of evolution, non-linear partial differential equations. For example, with Gibbon [21] he has recently derived a blow-up criterion for the 3D Euler equations in bounded domains

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that shows promise to be more easily implemented computationally. A computational investigation of possible blow-up for 3D Euler was carried out with Larios, Petersen, Titi and Wingate in [13]. This computational study is based on another, new blowup criterion, using the Euler-Voigt-α inviscid regularization.

Edriss Titi's many accomplishments have been recognized in different ways throughout his career. He was elected *Fellow of the Institute of Physics*, UK, in 2004, *Fellow of the Society for Industrial and Applied Mathematics (SIAM)* in 2012, and *Fellow of the Inaugural Class of the American Mathematical Society (AMS)* in 2013. Edriss Titi received the *Humboldt Research Award for Senior U.S. Scientists*, from the Alexander von Humboldt Stiftung/Foundation, Germany, in 2009, and the *Ciência sem Fronteiras - Science without Boundaries Scholarship*, by the Conselho Nacional de Desenvolvimento Científico (CNPq), Brazil, in 2013. In 2017 he was a *Senior Simons Professor* at the Polish Academy of Sciences, and in 2018 he will be the *Gaspard Monge Distinguished Visiting Professor* at École Polytechnique – Paris. He has also received the *Einstein Visiting Fellow* award, from the Einstein Stiftung/Foundation, Berlin, (2018–2020). In January 2018, he delivered the AMS Invited Address at the 2018 Mathematics Joint Meetings in San Diego. Most recently, in April of 2018, he was announced the winner of a *John Simon Guggenheim Memorial Foundation Fellowship*, the one and only fellow in the field of Applied Mathematics, for his past achievements and exceptional promise for future accomplishment.

In 2009, Titi and Cao received the *SIAM Activity Group on Analysis of Partial Differential Equations (SIAG/APDE) Prize*, which recognizes the authors of an outstanding article in Partial Differential Equations, for the paper titled "Global Well-Posedness of the Three-Dimensional Viscous Primitive Equations of Large Scale Ocean and Atmosphere Dynamic" published in Annals of Mathematics [22]. This work is a major contribution to the theory of the primitive equations, first derived by Richardson in the 1920s as a model of large-scale dynamics of the ocean and atmosphere. Short-time well-posedness was only established in the early 2000's, due to inherent difficulties with the lack of symmetry of the equations. The approach of Cao and Titi relies on a novel, more tractable formulation of the equations, which splits the velocity into baroclinic and barotropic modes. Later, in [23], Cao, Ibrahim, Nakanishi, and Titi proved also that for a certain class of initial data, the corresponding smooth solutions of the inviscid primitive equations blow up in finite time.

Edriss Titi's work continues to address important open problems in fluid mechanics and, more generally, in applied mathematics. Titi and Li [24] have recently proved the global well-posedness of a tropical atmosphere model with moisture, and justifed the relaxation limit associated with this model. In this work they settled the viscous version of an open problem proposed in an earlier work of Frierson, Majda and Pauluis in [25]. With Azouani and Olson, Titi has developed a feedback approach to the control of infinite-dimensional dissipative PDEs with finitely many controls, that is well adapted to data assimilation for weather prediction [26,27]. The scheme relies on the existence of finitely many determining modes, but does not require the existence of an inertial manifold or separation of scales. This novel idea has now been implemented in several context by Titi and his collaborators (see, e.g. [28–35] and references therein). In particular, Farhat, Lunasin, and Titi [36] have recently rigorously proved convergence of a particular data assimilation algorithm for the viscous 3D Planetary Geostrophic model. In this paradigm, they establish Charney's conjecture in [37] that all the state variables of the system can be determined by employing coarse spatial mesh measurements of the temperature alone.

Edriss Titi's many collaborations, at all levels, often spanning decades such as in the case of work with Constantin and Doering [38,39] are a testament to his energetic, enthusiastic, and above-all generous disposition. His collaboration with Foias since the 80's is particular poignant and has contributed new frameworks in the analysis of fluid mechanics (see, e.g. [40]). Recent works with Foias, Jolly, Kravchenko and Lithio [41,42], showcase a new framework for representing the long time dynamics of the 2D Navier–Stokes equations and other dissipative dynamical systems, in terms of an ODE which admits a Lyapunov function, defined in an infinite-dimensional phase space (see also [43–48] for its connection to the notion of determining modes, nodes, volume elements, etc.). The steady state set of this ODE is shown to be in one-to-one, bi-Lipschitz smooth correspondence with the set of full trajectories on the global attractor of the Navier–Stokes equations.

The 25 articles collected in this special volume cover a wide range of topics: well-posedness and regularity results for nonlinear models, reduced order models, long-time behavior, data assimilation, analysis and simulation of turbulence models and geophysical models, Onsager's conjecture and other applications. Specifically, Biswas et al. [49] study the well-posedness of the 3D Navier–Stokes equations in a new Gevrey-type class lying between the classical Gevrey classes and the scale of Sobolev spaces, while Bilgin and Kalantarov [50] investigate the existence of finitely many determining modes for the structurally damped nonlinear wave equation subject to homogeneous Dirichlet boundary condition in a bounded domain contained in $\mathbb{R}^3$. Chen et al. [51] examine the link between the equations of inviscid continuum mechanics and the motion of Riemannian manifolds, and Chepyzhov et al. [52] study convergence of global attractors for the two-dimensional damped and driven incompressible Navier–Stokes system with stress free boundary conditions in the vanishing viscosity limit.

Hu et al. [53] establish global well-posedness for the 2D Boussinesq equations with partial dissipation in bounded domains with the Navier type boundary conditions, while the paper of Blocher et al. [54] is devoted to the study the synchronization of chaotic systems using a nudging algorithm similar to [26] but considering the new problem of assimilation of noisy and time-averaged measurements in the Lorenz '63 system, in which the window of averaging may be unknown. Gibbon et al. [55] consider a model that couples the Navier–Stokes equations to the Cahn-Hilliard equations, and Anbarlooei et al. [56] extend Kolmogorov's K41 phenomenological theory of turbulence to obtain the Kolmogorov microscales for a family of non-Newtonian viscoplastic fluids. The paper by Seinen and Khouider [57] deals with the complexity of finding an accurate and efficient numerical solution for the sea-ice dynamics equations with a viscous-plastic rheology.

Bardos et al. [58] provide a justification of the Maxwell–Boltzmann approximation for electron density and prove that the reduced kinetic model for ions is globally well-posed, and Bodova et al. [59] study the Fokker–Planck equation derived in the large system limit of Markovian process describing the dynamics of quantitative traits. Camliyurt and Kukavica [60] address preservation of the Lagrangian analyticity radius of solutions to the Euler equations belonging to natural analytic space based on the size of Taylor (or Gevrey) coefficients, and Shvydkoy and Tadmor [61] extend their study of hydrodynamic models of self-organized evolution of agents with a singular interaction kernel.

Crisan and Holm [62] investigate wave breaking in a stochastic Camassa–Holm equation, while Doering and collaborators [63] study the long-time dynamics of a model of convection in the absence of buoyancy diffusion. Galochkina et al. [64] model blood clot growth as a traveling wave solution of the bistable reaction–diffusion system, and Ilyin et al. [65] analyze a hyperbolic relaxation of the classical Navier–Stokes problem in 2D bounded domain with Dirichlet boundary conditions.

Jiu et al. [66] prove global well-posedness of classical solutions to the 2D Cauchy problem for the compressible Navier–Stokes