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DETERMINING MODES FOR THE SURFACE QUASI-GEOSTROPHIC EQUATION

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ABSTRACT. We introduce a determining wavenumber for the surface quasi-geostrophic (SQG) equation defined for each individual trajectory and then study its dependence on the force. While in the subcritical and critical cases this wavenumber has a uniform upper bound, it may blow up when the equation is supercritical. A bound on the determining wavenumber provides determining modes, and measures the number of degrees of freedom of the flow, or resolution needed to describe a solution to the SQG equation.

This paper is dedicated to Professor Edriss S. Titi on the occasion of his sixtieth birthday with friendship and admiration.

KEY WORDS: Surface quasi-geostrophic equation, determining modes, global attractor, De Giorgi method.
CLASSIFICATION CODE: 35Q35, 37L30.

1. INTRODUCTION

In this paper we introduce a determining wavenumber $\Lambda_\theta(t)$ for the forced surface quasi-geostrophic (SQG) equation

$$\begin{aligned} \frac{\partial \theta}{\partial t} + u\cdot\nabla\theta + \nu\Lambda^\alpha\theta &= f, \\ u &= R^\perp\theta, \end{aligned} \tag{1.1}$$

on the torus $\mathbb{T}^2 = [0, L]^2$, where $0 < \alpha < 2$, $\nu > 0$, $\Lambda = \sqrt{-\Delta}$ is the Zygmund operator, and

$$R^\perp\theta = \Lambda^{-1}(-\partial_2\theta, \partial_1\theta).$$

The scalar function θ represents the potential temperature and the vector function u represents the fluid velocity. The initial data $\theta(0) \in L^2(\mathbb{T}^2)$ and the force $f \in L^p(\mathbb{T}^2)$ for some $p > 2/\alpha$ are assumed to have zero average.

The wavenumber $\Lambda_\theta(t)$ is defined solely based on the structure of the equation, but not on the force, regularity properties, or any known bounds on the solution. We prove that if two complete weak solutions $\theta_1, \theta_2 \in L^\infty((-\infty,\infty); L^2)$ (i.e., lying on the global attractor) coincide on frequencies below $\max\{\Lambda_{\theta_1}, \Lambda_{\theta_2}\}$, then $\theta_1 \equiv \theta_2$. While in the subcritical and critical cases this wavenumber has uniform upper bounds, it may blow up when the equation is supercritical. A bound on Λ_θ immediately provides determining modes, which in some sense measure the number of degrees of freedom of the flow, or resolution needed to describe a solution to the SQG equation.

The first result of finite dimensionality of a flow was obtained by Foias and Prodi for the 2D Navier-Stokes equations (NSE) in [26], where it was shown that low modes control high modes asymptotically as time goes to infinity. Then an explicit estimate on the number

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