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THE INVISCID LIMIT OF THE INCOMPRESSIBLE 3D NAVIER-STOKES EQUATIONS WITH HELICAL SYMMETRY

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ABSTRACT. In this paper, we are concerned with the vanishing viscosity problem for the three-dimensional Navier-Stokes equations with helical symmetry, in the whole space. We choose viscosity-dependent initial $\mathbf{u}_0^\nu$ with helical swirl, an analogue of the swirl component of axisymmetric flow, of magnitude $\mathcal{O}(\nu)$ in the L^2 norm; we assume $\mathbf{u}_0^\nu \to \mathbf{u}_0$ in H^1. The new ingredient in our analysis is a decomposition of helical vector fields, through which we obtain the required estimates.

KEY WORDS: Navier-Stokes equations; Euler equations; Helical symmetry; Vanishing viscosity limit.

2000 MATHEMATICS SUBJECT CLASSIFICATION. 76B47; 35Q30.

Dedicated to Edriss S. Titi, on the occasion of his 60^{th} birthday.

1. INTRODUCTION

The initial-value problem for the three-dimensional incompressible Navier-Stokes equations with viscosity $\nu > 0$ is given by

$$\begin{cases} \partial_t \mathbf{u}^\nu + \mathbf{u}^\nu \cdot \nabla \mathbf{u}^\nu + \nabla p^\nu = \nu \Delta \mathbf{u}^\nu & (\mathbf{x}, t) \in \mathbb{R}^3 \times (0, \infty), \\ \operatorname{div} \mathbf{u}^\nu = 0 & (\mathbf{x}, t) \in \mathbb{R}^3 \times (0, \infty), \\ \mathbf{u}^\nu(t = 0, \mathbf{x}) = \mathbf{u_0}^\nu & \mathbf{x} \in \mathbb{R}^3, \end{cases} \tag{1.1}$$

where $\mathbf{x} = (x, y, z)$, $\mathbf{u}^\nu = (u_1^\nu, u_2^\nu, u_3^\nu)$ is the velocity and p^ν is the pressure.

Formally, when $\nu = 0$, (1.1) becomes the classical incompressible Euler equations

$$\begin{cases} \partial_t \mathbf{u}^0 + \mathbf{u}^0 \cdot \nabla \mathbf{u}^0 + \nabla p^0 = 0 & (\mathbf{x}, t) \in \mathbb{R}^3 \times (0, \infty), \\ \operatorname{div} \mathbf{u}^0 = 0 & (\mathbf{x}, t) \in \mathbb{R}^3 \times (0, \infty), \\ \mathbf{u}^0(t = 0, \mathbf{x}) = \mathbf{u}_0 & \mathbf{x} \in \mathbb{R}^3. \end{cases} \tag{1.2}$$

Global existence of weak solutions and local in time well-posedness of strong solutions for problem (1.1) is due to J. Leray, see [12]. There is a vast literature on existence, uniqueness and regularity of solutions of (1.1),