Accepted Manuscript

Statistical solutions and Onsager's conjecture

U.S. Fjordholm, E. Wiedemann

PII: S0167-2789(17)30328-7
DOI: https://doi.org/10.1016/j.physd.2017.10.009
Reference: PHYSD 31961

To appear in: *Physica D*

Received date : 13 June 2017
Revised date : 12 October 2017
Accepted date : 19 October 2017

Please cite this article as: U.S. Fjordholm, E. Wiedemann, Statistical solutions and Onsager's conjecture, *Physica D* (2017), https://doi.org/10.1016/j.physd.2017.10.009

Statistical solutions and Onsager's conjecture

U. S. Fjordholm* E. Wiedemann†

October 12, 2017

Abstract

We prove a version of Onsager's conjecture on the conservation of energy for the incompressible Euler equations in the context of statistical solutions, as introduced recently by Fjordholm et al. [12]. As a byproduct, we also obtain an alternative proof for the conservative direction of Onsager's conjecture for weak solutions, under a weaker Besov-type regularity assumption than previously known.

Dedicated to Edriss S. Titi on the occasion of his 60th birthday.

1 Introduction

We consider the d-dimensional incompressible Euler equations: Find a function $v = (v^1, \ldots, v^d) : \mathbb{R}_+ \times D \to \mathbb{R}^d$ and a function $p : \mathbb{R}_+ \times D \to \mathbb{R}$ such that

$$\begin{aligned} \partial_t v + \sum_k \partial_{x^k}(vv^k) + \nabla p &= 0 \qquad x \in D,\ t > 0 \\ \nabla \cdot v &= 0 \qquad x \in D,\ t > 0 \\ v(0,x) &= v_0(x) \qquad x \in D. \end{aligned} \tag{1.1}$$

Here and below, the summation limits, when not specified, are always from $k = 1$ to $k = d$. The initial data v_0 is assumed to lie in $L^2(D)$. The spatial parameter x takes values in a set D, which we will take as either $\mathbb{R}^d$ or the (d-dimensional) torus $\mathbb{T}^d$ for simplicity[1]. The temporal domain is $[0,T]$ for some $T > 0$.

By a *solution of the Euler equations* we will mean a weak solution of (1.1), i.e. a function $v \in L^2_{loc}([0,T] \times D; \mathbb{R}^d)$ such that

$$\int_{\mathbb{R}_+} \int_D v\partial_t \varphi + \sum_k vv^k \partial_{x^k}\varphi + p\nabla\varphi \, dxdt + \int_D v_0(x)\varphi(0,x)\, dx = 0 \tag{1.2}$$

for all $\varphi \in C_c^\infty([0,T) \times \mathbb{R}^d)$, as well as satisfying the divergence free condition in the sense of distributions.

*Department of Mathematics, University of Oslo, Postboks 1053 Blindern, 0316 Oslo, Norway.
†Institut für Angewandte Mathematik, Leibniz Universität Hannover, Welfengarten 1, 30167 Hannover, Germany.

[1] On domains with boundaries, one can show the *local* version of the energy equality with almost no further effort, but in order to deduce from this also the *global* conservation of energy one requires some assumption of continuity at the boundary in addition to one of the usual Besov-type regularity assumptions. For a first result in this direction see [1].