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Statistical solutions and Onsager's conjecture

U. S. Fjordholm* E. Wiedemann†

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Abstract

We prove a version of Onsager's conjecture on the conservation of energy for the incompressible Euler equations in the context of statistical solutions, as introduced recently by Fjordholm et al. [12]. As a byproduct, we also obtain an alternative proof for the conservative direction of Onsager's conjecture for weak solutions, under a weaker Besov-type regularity assumption than previously known.

Dedicated to Edriss S. Titi on the occasion of his 60th birthday.

1 Introduction

We consider the d -dimensional incompressible Euler equations: Find a function $v = (v^1, \dots, v^d) : \mathbb{R}_+ \times D \rightarrow \mathbb{R}^d$ and a function $p : \mathbb{R}_+ \times D \rightarrow \mathbb{R}$ such that

$$\begin{aligned} \partial_t v + \sum_k \partial_{x^k} (v v^k) + \nabla p &= 0 & x \in D, \ t > 0 \\ \nabla \cdot v &= 0 & x \in D, \ t > 0 \\ v(0, x) &= v_0(x) & x \in D. \end{aligned} \tag{1.1}$$

Here and below, the summation limits, when not specified, are always from $k = 1$ to $k = d$. The initial data v_0 is assumed to lie in $L^2(D)$. The spatial parameter x takes values in a set D , which we will take as either \mathbb{R}^d or the (d -dimensional) torus \mathbb{T}^d for simplicity¹. The temporal domain is $[0, T]$ for some $T > 0$.

By a *solution of the Euler equations* we will mean a weak solution of (1.1), i.e. a function $v \in L^2_{loc}([0, T] \times D; \mathbb{R}^d)$ such that

$$\int_{\mathbb{R}_+} \int_D v \partial_t \varphi + \sum_k \int_D v v^k \partial_{x^k} \varphi + p \nabla \varphi \, dx dt + \int_D v_0(x) \varphi(0, x) \, dx = 0 \tag{1.2}$$

for all $\varphi \in C_c^\infty([0, T] \times \mathbb{R}^d)$, as well as satisfying the divergence free condition in the sense of distributions.

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¹On domains with boundaries, one can show the *local* version of the energy equality with almost no further effort, but in order to deduce from this also the *global* conservation of energy one requires some assumption of continuity at the boundary in addition to one of the usual Besov-type regularity assumptions. For a first result in this direction see [1].

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