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Exact Closed-Form Solutions of a Fully Nonlinear Asymptotic Two-Fluid Model

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Abstract

A fully nonlinear model of Choi and Camassa [1] describing one-dimensional incompressible dynamics of two non-mixing fluids in a horizontal channel, under a shallow water approximation, is considered. An equivalence transformation is presented, leading to a special dimensionless form of the system, involving a single dimensionless constant physical parameter, as opposed to five parameters present in the original model. A first-order dimensionless ordinary differential equation describing traveling wave solutions is analyzed. Several multi-parameter families of physically meaningful exact closed-form solutions of the two-fluid model are derived, corresponding to periodic, solitary, and kink-type bidirectional traveling waves; specific examples are given, and properties of the exact solutions are analyzed.

1 Introduction

Over the years, in order to describe specific physical settings, such as, for example, surface and internal waves, multiple simplified models of the systems of Euler and Navier-Stokes fluid dynamics equations have been derived, aiming at the reduction of the mathematical complexity of the full set of equations, while retaining essential properties of phenomena of interest and providing sufficient physical insight and computational precision. Basic examples of such simplifications include dimension reductions, linearizations, and more general approximations involving asymptotic relationships. Fundamental nonlinear partial differential equations (PDEs) of mathematical physics, such as Burgers', Korteweg-de Vries (KdV), nonlinear Schrödinger, and Kadomtsev-Petviashvili (KP) equations, as well as many other important models like shallow water equations, Camassa-Holm and Degasperis-Procesi (DP) equations, arise in the context of fluid dynamics. Importantly, such reduced models were often found to exhibit rich mathematical structure, such as integrability, Hamiltonian structure, existence of infinite hierarchies of conservation laws, and solutions in the form of single and/or multiple nonlinear solitary waves (solitons, peakons, etc.). In many cases, exact solutions of reduced models correspond to, and in fact closely describe, physical phenomena. Examples are provided by solitary wave solutions of the KdV equation modeling long waves in shallow channels, periodic solutions of the KP equa-

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