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Synchronisation Under Shocks: the Lévy Kuramoto Model

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Abstract

We study the Kuramoto model of identical oscillators on Erdős-Rényi (ER) and Barabasi-Alberts (BA) scale free networks examining the dynamics when perturbed by a Lévy noise. Lévy noise exhibits heavier tails than Gaussian while allowing for their tempering in a controlled manner. This allows us to understand how 'shocks' influence individual oscillator and collective system behaviour of a paradigmatic complex system. Skewed α-stable Lévy noise, equivalent to fractional diffusion perturbations, are considered, but overlaid by exponential tempering of rate λ. In an earlier paper we found that synchrony takes a variety of forms for identical Kuramoto oscillators subject to stable Lévy noise, not seen for the Gaussian case, and changing with α: a noise-induced drift, a smooth α dependence of the point of cross-over of synchronisation point of ER and BA networks, and a severe loss of synchronisation at low values of α. In the presence of tempering we observe both analytically and numerically a dramatic change to the $\alpha < 1$ behaviour where synchronisation is sustained over a larger range of values of the 'noise strength' σ, improved compared to the $\alpha > 1$ tempered cases. Analytically we study the system close to the phase synchronised fixed point and solve the tempered fractional Fokker-Planck equation. There we observe that densities show stronger support in the basin of attraction at low α for fixed coupling, σ and tempering λ. We then perform numerical simulations for networks of size $N = 1000$ and average degree $\bar{d} = 10$. There, we compute the order parameter r as a function of σ for fixed α and λ and observe values of $r \approx 1$ over larger ranges of σ for $\alpha < 1$ and $\lambda \neq 0$. In addition we observe drift of both positive and negative slopes for different α and λ when native frequencies are equal, and confirm a sustainment of synchronisation down to low values of α. We propose a mechanism for this in terms of the basic shape of the tempered stable Lévy densities for various α and how it feeds into Kuramoto oscillator dynamics and illustrate this with examples of specific paths.

1. Introduction

The dynamical system of oscillators proposed by Kuramoto [1] provides a way to understand some of the complexities of spontaneous synchronisation inherent to some real-world complex systems of physical, biological, or social nature. The original model captures structure and rhythmic behaviour. Generalising the all-to-all case to non-complete networks [2, 3, 4, 5] and subjecting oscillators to noisy Gaussian perturbations [2, 6, 7, 8] or heavier-tailed perturbations [9] offers further applicability.

In this paper we examine the relationship between synchronisation of oscillators and the speed of decay in the tails of the noise distribution that seeks to disrupt their collective behaviour. As such, we perturb each oscillators by a Lévy process, a natural generalisation of Brownian motion, that allows heavier tails in the distribution of the process increments. More precisely, we use a tempered stable process so that we can control the rate of tail decay to demonstrate a striking relationship between this rate and the onset of synchronisation within the system.

The main motivation of this study lies in the viability of a Lévy Kuramoto model as a representation of social processes or distributed decision-making [10, 11, 12, 13]. Human cognitive processes are known to be cyclic [14], noisy [15] and involve leaps of intuition in complex problem solving [16] or recognition of previously experienced patterns, known as 'priming' [17, 18]. Modelling these leaps by a Lévy process enables quantification of these otherwise qualitative models. Additionally, recent results in the field