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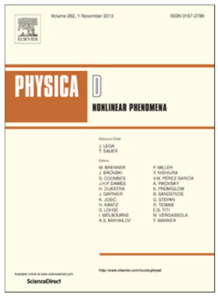

Inverse scattering transform and soliton solutions for square matrix nonlinear Schrödinger equations with non-zero boundary conditions

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Inverse scattering transform and soliton solutions for square matrix nonlinear Schrödinger equations with non-zero boundary conditions

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Abstract

The inverse scattering transform (IST) with non-zero boundary conditions at infinity is developed for an $m \times m$ matrix nonlinear Schrödinger-type equation which, in the case $m = 2$, has been proposed as a model to describe hyperfine spin $F = 1$ spinor Bose-Einstein condensates with either repulsive interatomic interactions and anti-ferromagnetic spin-exchange interactions (self-defocusing case), or attractive interatomic interactions and ferromagnetic spin-exchange interactions (self-focusing case). The IST for this system was first presented by Ieda, Uchiyama and Wadati in [55], using a different approach. In our formulation, both the direct and the inverse problems are posed in terms of a suitable uniformization variable which allows to develop the IST on the standard complex plane, instead of a two-sheeted Riemann surface or the cut plane with discontinuities along the cuts. Analyticity of the scattering eigenfunctions and scattering data, symmetries, properties of the discrete spectrum, and asymptotics are derived. The inverse problem is posed as a Riemann-Hilbert problem for the eigenfunctions, and the reconstruction formula of the potential in terms of eigenfunctions and scattering data is provided. In addition, the general behavior of the soliton solutions is analyzed in details in the 2×2 self-focusing case, including some special solutions not previously discussed in the literature.

1 Introduction

In recent years there has been an increasing focus on the study of multicomponent Bose-Einstein condensates (BECs), and particularly spinor condensates, within the context of atomic and nonlinear wave physics. Multicomponent ultracold atomic gases and BECs may be composed by two or more atomic gases, and may have the form of various different mixtures, which have been observed in experiments [43]. Such multicomponent systems support various types of matter-wave soliton complexes, with the type of soliton in one species being the same or different to that in the other species. Unlike what happens in multicomponent nonlinear optics, where Kerr-type