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Interaction of non-radially symmetric camphor particles

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H I G H L I G H T S

- Interaction between two non-radially symmetric camphor particles is investigated.
- ODEs for the positions and angles are obtained by the center manifold reduction.
- Major axes of two elliptic particles align vertically to the center-to-center line.
- Theoretical results are supported by numerical and experimental results.

A R T I C L E I N F O

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A B S T R A C T

In this study, the interaction between two non-radially symmetric camphor particles is theoretically investigated and the equation describing the motion is derived as an ordinary differential system for the locations and the rotations. In particular, slightly modified non-radially symmetric cases from radial symmetry are extensively investigated and explicit motions are obtained. For example, it is theoretically shown that elliptically deformed camphor particles interact so as to be parallel with major axes. Such predicted motions are also checked by real experiments and numerical simulations.

1. Introduction

Spontaneous motions are one of the most attractive behaviors both from phenomenological and theoretical points of view (e.g. [1–3] and the references in them). Traveling pulses are typical examples and extensive research has been conducted in this area (e.g. [1,4–7]).

Recently, a camphor particle floating on water was considered and the occurrence of spontaneous motions of a camphor particle was reported theoretically and experimentally [8–18]. Among them, radially symmetric camphor particles have been mainly treated and it was shown that the spontaneous motion can occur due to the symmetry breaking of the profile of dissolved camphor molecules at the water surface even if the camphor particle is radially symmetric. In practice, such symmetry breaking of camphor concentration profile causes an asymmetrical surface tension around the particle and, therefore, spontaneous motions of a camphor particle appear as the bifurcating branch from a standing solution by a pitchfork bifurcation. This can be regarded as a typical example of traveling spot in $\mathbb{R}^2$.

On the other hand, an asymmetric camphor particle has been considered as a natural extension from symmetric particles. It was reported that for an elliptic camphor particle the spontaneous traveling motion in the minor-axis direction primarily appears both theoretically and numerically (e.g. [14,15]). They also investigated various parameter sets numerically and checked what kind of behaviors, such as a traveling motion, self-rotating motion, and so on, appear in each parameter region. In particular, they suggested the existence of an appropriate parameter region in which an elliptic camphor particle can stand still.

In this study, we focus on the aforementioned fact and investigate the interaction between two standing asymmetric camphor particles as the first step to analyze multi-camphor particles. In fact, there has been a lot of research related to multi-camphor particles (e.g. [19–28]) such as analysis of the jam of camphor particles on a circle (e.g. [28]) although almost all of them are related to the interaction in one-dimensional space.

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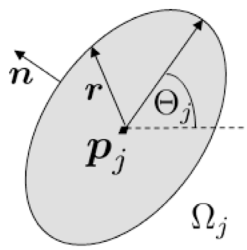

Fig. 1. Camphor particle Ω_j with the location $\boldsymbol{p}_j$ and the characteristic angle Θ_j.

On the other hand, only few analyzes of the interaction between multi-camphor particles in two-dimensional space have been presented, and even so, only for the case of radially symmetric particles. In fact, in [13], the interaction between two radially symmetric camphor particles was shown to be repulsive in a mathematically rigorous way, in which the equation describing the motion of two interacting traveling camphor particles with sufficiently small velocities was derived.

Here, we consider the interaction between non-radially symmetric camphor particles, but use the interaction of standing spot solutions as the first step to deal with multi-camphor particles with non-radially symmetric shapes.

We only consider two camphor particles. The considered model equation is

$$\begin{cases} \tau_1 \dot{\boldsymbol{p}}_j &= \displaystyle\int_{\partial\Omega_j(t)} \gamma(u)\boldsymbol{n}\, ds, \\ \tau_2 \dot{\Theta}_j &= \displaystyle\int_{\partial\Omega_j(t)} \gamma(u)(\boldsymbol{r}\times\boldsymbol{n})\, ds, \\ \partial_t u &= d\Delta u - \alpha u + a_0\left(\chi_{\Omega_1(t)}(\boldsymbol{x}) + \chi_{\Omega_2(t)}(\boldsymbol{x})\right) \end{cases} \tag{1.1}$$

for $j = 1, 2$, where all coefficients τ_j, d, α, and a_0 are all positive constants. $\gamma(u)$ is a decreasing function, and $\chi_A(\boldsymbol{x})$ is the characteristic function for a set $A \subset \mathbb{R}^2$. Here, $u(t, \boldsymbol{x})$ denotes the surface concentration of the camphor molecular layer on water surface. Camphor particles are represented by $\Omega_j(t)$ with $\partial\Omega_j(t) = \{\boldsymbol{x} \in \mathbb{R}^2;\ \boldsymbol{x} = \boldsymbol{p}_j(t) + \boldsymbol{r}\}$ as in Fig. 1, where $\boldsymbol{n}$ is the outward normal unit vector of $\partial\Omega_j(t)$, $\boldsymbol{p}_j$ denote the locations of camphor particles, and Θ_j are the characteristic angles of the camphor particles. The vector product in two dimensions is defined as $\boldsymbol{r}\times\boldsymbol{n} = r_1n_2 - r_2n_1$, where $\boldsymbol{r} = {}^t(r_1, r_2)$ and $\boldsymbol{n} = {}^t(n_1, n_2)$.

The physical meanings of Eq. (1.1) are as follows. The first and second equations represent balanced equations for the positions and the characteristic angles of camphor particles, respectively. The left-hand sides of both equations denote the resistance for translation and rotation, while the right-hand sides are the force and torque originating from the inhomogeneity of surface tension $\gamma(u)$. The constants τ_1 and τ_2 are the resistance coefficients for the translation and the rotation, respectively. The third equation represents the time evolution of concentration of camphor molecular layer, and the right-hand side is composed of the terms describing diffusion, sublimation into the air, and supply from the camphor particle. Here, d, α, and a_0 are the diffusion constant, the sublimation rate, and supply rate from the camphor particle per unit area, respectively.

In real systems, the Marangoni convection is induced by the surface tension difference at water surface [29–32]. Therefore, we have to consider hydrodynamics to discuss the actual phenomenon in more detail. In the other place, one of the authors discuss that the effect of the hydrodynamic effect is regarded as effective diffusion when the camphor particle is moving with a sufficiently small velocity [33]. In this sense, our model does not precisely describe the motion of the camphor particle. However, since our model is constructed with simple equations, and thus we consider our model can be adopted to the other phenomena such as chemotactic motion in which an object release chemicals from which the object tends to escape [1,34,35].

For a single camphor particle with elliptic shape, the model equation was originally proposed in [15] as follows:

$$\begin{cases} m\ddot{\boldsymbol{p}} + \tau_1\dot{\boldsymbol{p}} &= \displaystyle\int_{\partial\Omega(t)} \gamma(u)\boldsymbol{n}\, ds, \\ I\ddot{\Theta} + \tau_2\dot{\Theta} &= \displaystyle\int_{\partial\Omega(t)} \gamma(u)(\boldsymbol{r}\times\boldsymbol{n})\, ds, \\ \partial_t u &= d\Delta u - \alpha u + a_0\chi_{\Omega(t)}(\boldsymbol{x}), \end{cases} \tag{1.2}$$

where $m\ddot{\boldsymbol{p}}$ and $I\ddot{\Theta}$ are inertia terms for translation and rotation, respectively. As mentioned above, it was shown in [14,15] that an elliptic camphor particle tends to move in the minor-axis direction. It was also suggested by numerical simulations that a single elliptic camphor particle stands still when τ_1 and τ_2 are large. In this paper, we consider this case but with two identical elliptic camphors. In that case, each camphor particle does not move if it exists alone and hence, we can expect that the movements of two interacting camphor particles are rather slow. Thus, we neglect acceleration terms $\ddot{\boldsymbol{p}}$ and $\ddot{\Theta}$ in (1.2) and consider (1.1).

By analyzing the interaction of them in (1.1), we can show that each camphor particle with elliptic shape moves and rotates so that their major axes are orthogonal to the center line connecting $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$ if each camphor particle is slightly deformed from radial symmetry.

This paper is organized as follows: In Section 2, we present the model equation for a single particle that we treat in this paper precisely. It is not (1.1) but a slightly modified version. In Section 3, the analysis of the interaction between two camphor particles is described. The validation of theoretical results using real experiments is described in Section 4. The theoretical results are also checked with numerical calculation and subsequently presented in Section 5.

2. Preliminaries for (1.1) and a single camphor particle

Before we deal with two camphor particles, we first consider a model of a single camphor particle. The model equation for a single camphor particle corresponding to (1.1) is

$$\begin{cases} \tau_1\dot{\boldsymbol{p}} &= \displaystyle\int_{\partial\Omega(t)} \gamma(u)\boldsymbol{n}\, ds, \\ \tau_2\dot{\Theta} &= \displaystyle\int_{\partial\Omega(t)} \gamma(u)(\boldsymbol{r}\times\boldsymbol{n})\, ds, \\ \partial_t u &= d\Delta u - \alpha u + a_0\chi_{\Omega(t)}(\boldsymbol{x}). \end{cases} \tag{2.1}$$

Dividing the first and the second equations of (2.1) by τ_1 and τ_2, respectively, we consider

$$\begin{cases} \dot{\boldsymbol{p}} &= \displaystyle\int_{\partial\Omega(t)} \gamma_1(u)\boldsymbol{n}\, ds, \\ \dot{\Theta} &= \displaystyle\int_{\partial\Omega(t)} \gamma_2(u)(\boldsymbol{r}\times\boldsymbol{n})\, ds, \\ \partial_t u &= d\Delta u - \alpha u + a_0\chi_{\Omega(t)}(\boldsymbol{x}). \end{cases} \tag{2.2}$$

Hereafter we assume γ_1 and γ_2 are certain decreasing functions but make no other assumptions, that is, $\gamma_j(u)$ are not necessarily equal to $\gamma(u)/\tau_j$ of (1.1).

Let $R(\Theta) := \begin{pmatrix} \cos\Theta & -\sin\Theta \\ \sin\Theta & \cos\Theta \end{pmatrix}$ and Ω_0 be a fixed camphor particle located at the origin O and the characteristic angle $\Theta = 0$ as in Fig. 2. Then Ω with the location $\boldsymbol{p}$ and the characteristic angle Θ is