Physica D 365 (2018) 27–41

Contents lists available at ScienceDirect

Physica D

journal homepage: www.elsevier.com/locate/physd

Wave propagation in a strongly nonlinear locally resonant granular crystal

K. Vorotnikov [a,*], Y. Starosvetsky [a], G. Theocharis [b], P.G. Kevrekidis [c]

[a] Faculty of Mechanical Engineering, Technion Israel Institute of Technology, Technion City, Haifa 32000, Israel
[b] Laboratoire d'Acoustique de l'Université du Maine, UMR-CNRS 6613, Av. Olivier Messiaen, Le Mans 72000, France
[c] Department of Mathematics and Statistics, University of Massachusetts, Amherst, MA 01003-4515, USA

ARTICLE INFO

Article history:
Received 25 September 2017
Accepted 9 October 2017
Available online 23 November 2017
Communicated by V.M. Perez-Garcia

Keywords:
Granular chains
Solitary waves
Nanoptera
Mass-in-mass systems
Locally resonant crystals

ABSTRACT

In this work, we study the wave propagation in a recently proposed acoustic structure, the locally resonant granular crystal. This structure is composed of a one-dimensional granular crystal of hollow spherical particles in contact, containing linear resonators. The relevant model is presented and examined through a combination of analytical approximations (based on ODE and nonlinear map analysis) and of numerical results. The generic dynamics of the system involves a degradation of the well-known traveling pulse of the standard Hertzian chain of elastic beads. Nevertheless, the present system is richer, in that as the primary pulse decays, secondary ones emerge and eventually interfere with it creating modulated wavetrains. Remarkably, upon suitable choices of parameters, this interference "distills" a weakly nonlocal solitary wave (a "nanopteron"). This motivates the consideration of such nonlinear structures through a separate Fourier space technique, whose results suggest the existence of such entities not only with a single-side tail, but also with periodic tails on both ends. These tails are found to oscillate with the intrinsic oscillation frequency of the out-of-phase motion between the outer hollow bead and its internal linear attachment.

1. Introduction

Dynamics of one-dimensional granular chains has attracted substantial interest from the researchers of quite different scientific areas [1–24] due to their exciting dynamical properties. These chains support the formation of highly robust, strongly localized and genuinely traveling elastic stress waves. The existence of traveling waves was originally proved in [7] using the variational approach of [25], yet no information was given on their profile. Their single pulse character (in the strain variables) was rigorously shown in [26], following the approach of [27], and the doubly exponential character of their spatial decay in the absence of precompression was established. Earlier work on the basis of long wavelength approximations and numerical computations had conjectured that the waves were genuinely compact (spanning a finite number of elements) [1].

Recent studies [9–18] in the area have been mainly concerned with the effect of various types of structural inhomogeneities induced in the granular chain. The latter leads to a modulation of the solitary waves, as well as to new kinds of breathing modes produced either robustly [12–14,28,29] or transiently [30]. Wave propagation in tapered and decorated granular chains has been extensively studied in [9–11] both analytically and numerically. The approximations developed in these works for the estimation of the maximal pulse velocity recorded on each one of the granules along with its propagation through such inhomogeneous granular chains have demonstrated a good correspondence of the analytic predictions with the numerical simulations. Additional experimental, computational and analytical studies were devoted to the dynamics of the periodic granular chains (e.g. diatomic chains, granular containers, etc.) under various conditions of initial precompression [12–18]. Dynamics of primary pulses in the non-compressed granular chain perturbed by a weak dissipation has been considered in [19–24]. These, in turn, shed light on the evolution of the primary pulses in the dissipative, 1D granular media and provide some qualitative theoretical (in some cases in connection with experiments [23]) estimations for modeling the dissipation in the chain as well as depicting the rate of decay of the primary pulse. A systematic theoretical attempt to capture the (decaying) evolution of a primary pulse in the granular chain subject to on-site perturbation has been provided in the extensive study of [31].

In the present paper we study a novel acoustic structure which has been recently considered in some experimental and theoretical studies [32–34], the locally resonant granular crystal. The fundamental unit cell of these periodic systems is made of an outer mass

* Corresponding author.
E-mail address: kirill.vorotnikov@inria.fr (K. Vorotnikov).

https://doi.org/10.1016/j.physd.2017.10.007
0167-2789/

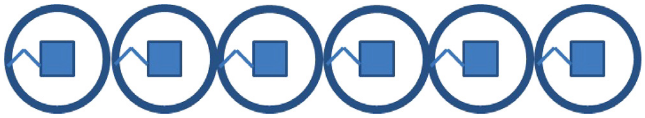

Fig. 1. Scheme of the model under consideration.

(hollow spherical shell and an inner mass connected by a linear spring). Our principal aim is to examine the dynamics of this novel class of chains, as regards their ability to propagate traveling waves in the presence of these internal resonators. What we generally observe in this setting is a decay of the principal pulse (in the strain variables), due to its coupling to the internal variables. The rate of such decay of a primary pulse can be fully captured analytically. Perhaps even more importantly, we show that, depending on the parameters of the internal resonator (i.e. coupling stiffness and mass), the response can range from the above mentioned (continuous) decay, to the formation of a genuinely traveling primary pulse. The latter scenario is examined in close detail and is identified as a case example of a "nanopteron" solution, whose tail carries an oscillation with the intrinsic linear frequency of the system (of the relative motion of the outer and inner mass). To the best of our knowledge, and although such weakly nonlocal solitary waves have been studied extensively in a series of examples in physical sciences and engineering [35], this is only the second example of the reporting of such a nanopteronic solution in granular systems. The potential observation of such solutions in granular systems has been earlier suggested based on the numerical observations of [29], while this possibility has been proposed theoretically a decade ago for FPU lattices in the work of [36]. This motivates us to further examine the problem using the methodology of [27]. As a result of this study we illustrate that it is possible in fact to produce nanoptera with oscillating tails on both ends of the principal pulse. These results pave the way for a previously unexplored class of solutions in these "mass-in-mass" systems and render them especially intriguing candidates for experimental investigations.

Our presentation is structured as follows. In Section 2, we offer the basic setup and the corresponding theoretical model. In Section 3, we provide the analytical approach that captures the typical (and systematic) decay of a primary pulse in the "mass-in-mass" (hereafter abbreviated as MiM) system. In Section 4, we test these predictions numerically, obtaining good agreement with the analytical predictions, but also shedding light on how a nanopteronic solution can be seen to spontaneously emerge. In Section 5, we propose a different analytical–numerical approach for capturing such solutions and offer a proof-of-principle confirmation of their existence (with tails on both sides of the principal pulse). Finally, in Section 6, we summarize our results and present our conclusions, as well as a number of directions for future study. Lastly, in the appendix we provide some technical details about the form of the traveling wave in the homogeneous granular chain that are used in our theoretical approach for capturing the decay of the primary pulse in the MiM setting.

2. Physical model

In the present study we consider the uncompressed, one-dimensional, locally resonant granular crystal composed of hollow elastic spheres in contact, containing linear resonators, as this is illustrated in Fig. 1. According to [37], the contact interaction of two hollow spheres depends strongly on the thickness of the spherical shells. However, for relatively thick spherical shells, the interaction contact follows the Hertzian contact law [1].

The governing equations of motion can then be written as follows,

$$
\begin{aligned}
M_i \frac{d^2 U_i}{dt^2} &= \left(\frac{4}{3}\right) E^* \sqrt{R_i}\left[(U_{i-1}-U_i)_+^{3/2} - (U_i - U_{i+1})_+^{3/2}\right] \\
&\quad + k\,(u_i - U_i)\,,\ \forall i,\ i \in N \\
m_i \frac{d^2 u_i}{dt^2} &= -k\,(u_i - U_i)\,.
\end{aligned} \tag{1}
$$

Here U_i is the displacement of the ith sphere, while u_i is the displacement of the small mass, linearly coupled attachment inside the ith sphere, r_i is the radius of the sphere, M_i is the mass of the sphere; and $E^* = E/2\left(1-\mu^2\right)$; E is the elastic (Young's) modulus and μ is the Poisson's ratio of the sphere. We note that the interaction force between the neighboring elements is given by $F = (4/3)\,E^* \sqrt{\frac{R_i}{2}}\,\Delta^{3/2}$, where R_i is bead radius (assumed implicitly to be uniform in the above expression i.e., independent of i) and Δ is their relative displacement. Moreover, the $(+)$ subscripts in (1) and (3) indicate that only non-negative values of the expressions in parentheses are considered, i.e., the interaction is tensionless. We should also mention in passing that a mathematically similar system with (isolated) external resonators (a so-called mass-with-mass setting) has recently been proposed in [38]. It is noteworthy that despite the mathematical equivalence of these two settings, their experimental realizations are quite different.

The system nondimensionalization is performed as follows

$$
\begin{aligned}
&X_i = \frac{U_i}{R_i};\ x_i = \frac{u_i}{R_i};\ \tau = \left[\frac{E^*}{\sqrt{2}\pi R_i^2 \rho}\right]^{1/2} t;\ \widetilde{\kappa} = \frac{3\sqrt{2}k}{4R_i E^*}; \\
&\nu = \frac{m_i}{M_i} = \frac{m}{M}.
\end{aligned} \tag{2}
$$

It is important to note that in the present study we assume (with one caveat to be explained below) that the outer and inner masses are uniform all through the chain (i.e. $R_i = R$, $M_i = M$, $m_i = m$).

Substituting (2) into (1) we end up with the following, non-dimensional set of equations governing the system dynamics

$$
\begin{aligned}
X_{i,\tau\tau} &= \left[(X_{i-1}-X_i)_+^{3/2} - (X_i - X_{i+1})_+^{3/2}\right] + \widetilde{\kappa}\,(x_i - X_i) \\
\nu x_{i,\tau\tau} &= -\widetilde{\kappa}\,(x_i - X_i)\,.
\end{aligned} \tag{3}
$$

To make the further analysis of (3) somewhat simpler it is convenient to introduce the coordinates of relative displacements (i.e., strains) for both outer and inner masses,

$$
\Delta_i = X_i - X_{i+1} \tag{4}
$$

$$
d_i = x_i - x_{i+1}. \tag{5}
$$

Substituting (4) and (5) into (3) we obtain the following set of equations,

$$
\begin{aligned}
\Delta_{i,\tau\tau} &= \Delta_{(i-1),+}^{3/2} - 2\Delta_{(i),+}^{3/2} + \Delta_{(i+1),+}^{3/2} + \widetilde{\kappa}\,(d_i - \Delta_i) \\
\nu d_{i,\tau\tau} &= -\widetilde{\kappa}\,(d_i - \Delta_i)\,.
\end{aligned} \tag{6}
$$

The goal of the present study is to examine the wave propagation along the contacts of the outer masses (which contribute to highly nonlinear dynamics) in the presence of the internal mass attachments. We assume that the coupling between the internal mass and the outer sphere is linear and weak such that $\widetilde{\kappa}$ is treated as a small system parameter $\widetilde{\kappa} = \varepsilon$. We anticipate that as the primary pulse propagates down the chain, it is possible for it to "transfer" energy to the internal, linear attachments, storing it in the form of potential energy and thus depriving the original pulse from its initial kinetic energy. This, in turn, is expected to yield a decay of the amplitude of the primary pulse, which we now consider in three distinct asymptotic limits, namely: (1) $\nu \ll 1$, (2) $\nu = O(1)$ and (3) $\nu \gg 1$.