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Homogenization of one-dimensional draining through heterogeneous porous media including higher-order approximations

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HIGHLIGHTS

- Homogenization theory is applied to draining of a heterogeneous saturated porous media.
- The position as a function of time of a planar free surface is predicted.
- Higher-order approximations are obtained and compared with the solution of the full problem.
- Porous media with smoothly varying permeability as well as porous media with discrete layers are considered.

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ABSTRACT

We examine a mathematical model of one-dimensional draining of a fluid through a periodically-layered porous medium. A porous medium, initially saturated with a fluid of a high density is assumed to drain out the bottom of the porous medium with a second lighter fluid replacing the draining fluid. We assume that the draining layer is sufficiently dense that the dynamics of the lighter fluid can be neglected with respect to the dynamics of the heavier draining fluid and that the height of the draining fluid, represented as a free boundary in the model, evolves in time. In this context, we neglect interfacial tension effects at the boundary between the two fluids. We show that this problem admits an exact solution. Our primary objective is to develop a homogenization theory in which we find not only leading-order, or effective, trends but also capture higher-order corrections to these effective draining rates. The approximate solution obtained by this homogenization theory is compared to the exact solution for two cases: (1) the permeability of the porous medium varies smoothly but rapidly and (2) the permeability varies as a piecewise constant function representing discrete layers of alternating high/low permeability. In both cases we are able to show that the corrections in the homogenization theory accurately predict the position of the free boundary moving through the porous medium.

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1. Introduction

Fluid transport in subsurface systems is important in many environmental and geophysical applications. These are complex environments, often not easily accessible, in which multiple physical, chemical and biological processes can take place. These types of porous media may involve structures that can be idealized as layers, correlated random fields, or facies. Because quantitative evaluations of the state of these complex systems are desired, mathematical models based upon conservation principles are routinely used to meet this objective.

Groundwater is a vital resource heavily relied upon as a source of drinking water. Contamination of such sites requires the implementation of well-thought-out remediation strategies. For example, in the case of contamination by dense nonaqueous phase liquids (DNAPLs), Miller and coworkers [1–3] have proposed remediation strategies that introduce dense brine into the subsurface system along with chemical surfactants to mobilize the DNAPLs and enable efficient removal. Another industrial/environmental application involves the sequestration of CO_2 in carefully-selected underground formations in an effort to reduce atmospheric emissions (e.g. the Sleipner Field [4,5]). Simply put, this procedure involves pumping the relatively light CO_2 into the subsurface system and releasing it beneath an impermeable caprock layer or between two such caprock layers. Other mathematical research is driven by the energy sector for applications related to oil recovery [6] and

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hydraulic fracturing, in which complex solutions are injected into tight shale deposits under high pressures to create relatively high permeability fractures that allow the efficient recovery of existing natural gas reserves.

In these wide-ranging applications gravity-driven motion is often a dominant mechanism for transport and the subsurface system properties are in general heterogeneous. Thus, mathematical and computational models of gravity currents in porous media have received much attention, particularly in the area of CO_2 sequestration. A wide range of issues have been addressed in this context, including the CO_2 injection problem [7,8], gravity current flows with leakage due to a fractured substrate [9], localized faults [10–14], leakage between confined aquifers [15] and leakage at the edge of a porous reservoir [16]. Still other important issues include two-phase transport [17], convective dissolution of CO_2 [18], inclination of the impermeable layer [13,19,20], gravity currents and dynamical influence of confinement and topographic features [20–24], gravity current dynamics in layered porous media [25–28], capillary/pinning effects [29–31], and residual trapping [20,28]. For an excellent review article on the fluid mechanics of CO_2 storage see Huppert & Neufeld [32]. Fundamentals of gravity currents in porous media can be found in the articles by Huppert and coworkers [33,34]. Evolving methods for modeling two-fluid-phase porous medium flow are both scale and thermodynamically consistent [35], while beyond the scope of the models considered herein.

In natural settings of interest for CO_2 sequestration, oil recovery, for example, it is common to encounter spatially-varying permeability and these issues have been explored in a number of references (e.g. [6,25–28]). Analysis and computation for random media has also received much attention (e.g. [36–40]) including situations of flow with non-Newtonian fluids [41]. Other studies have explored gravity-driven flow in the presence of power law type dependence of the permeability in the vertical direction (aligned with gravity) [16,33,42,43] as well as in the horizontal direction (perpendicular to gravity) [44]. Pritchard, Woods and Hogg [45] studied the problem of a gravity current in a porous media flowing over and draining through an underlying, thin, low permeability layer which in turn was situated above another deep permeable layer. Horizontal flow as well as vertical flow through a porous 'substrate' was of interest. Their configuration was similar to the flow of a viscous gravity current over a porous media [46,47] or the capillary spreading problem on a porous substrate by Davis & Hocking [48,49]. Periodically-layered media bounded below by an impermeable layer has also been explored in the context of gravity currents and homogenization theory used to obtain effective values of permeability in vertically, horizontally and two-dimensionally periodic structures [50–53].

There are many critical aspects associated with transport phenomena occurring in applications of groundwater remediation, CO_2 sequestration, hydraulic fracturing and oil recovery (e.g. capillary trapping, precipitation, dissolution reaction, multiphase processes, fluid instabilities, …). The potential for these applications to arise in heterogeneous porous media and involve the prediction of free-boundary motion provides a general motivation for the present study. In order to focus on this specific issue – free boundary motion in heterogeneous porous media – we address a model simplified both from a physical and a geometrical point of view.

We are also motivated by previous work on homogenization theory applied to study free-surface flows in layered porous media. In particular, one of the problems explored by Anderson, McLaughlin and Miller [50] involved a two-dimensional gravity current propagating through porous media whose permeability was periodically-varying in the direction perpendicular to gravity—a configuration opposite to that of layering typically encountered in practice. Homogenization theory (e.g. [54,55]) in that case showed that a leading-order theory provided an effective permeability that characterized the average motion of the gravity current. Anderson et al. [50] also obtained corrections to this leading-order theory. These results showed that higher-order agreement between the homogenization results and full numerical simulations could be obtained but that the corrections were not valid at the leading front (contact line) of the gravity current. That discrepancy between the homogenization theory and the numerical simulations led us to explore the idealized one-dimensional problem that is the subject of the present paper. In particular, we consider the situation of one dimensional draining of a saturated porous material whose permeability varies rapidly in the direction of the flow. Here the flow is unidirectional and the draining of a finite amount of fluid in an initially saturated porous medium is gravity driven. We note that a different porous media draining problem, in which a two-dimensional slumping gravity current is allowed to drain out of one horizontal boundary has been examined by Mathunjwa and Hogg [56]. For our one-dimensional problem we develop homogenization theory results that address not only leading-order averaging but also higher order corrections associated with the rapid permeability variation. We compare these results to numerical simulations of the full problem and show that the homogenization theory correctly captures the details of the front propagation. In doing so our aim is that our homogenization theory establishes a framework upon which to build improved approximations for more complicated geometries and flows.

Our paper is organized as follows. In Section 2 we describe the governing equations for porous media flow through 'horizontal' layers (permeability varies in the direction of gravity) and the direct (exact) solution that can be expressed implicitly in terms of integrals of a prescribed permeability function. In Section 3 we outline a homogenization procedure that extends the classical ones through the use of a strained coordinate expansion (or near-identity transformation) to address the motion of the free surface. In Section 4 we show comparisons of the exact and approximate solutions to demonstrate that the homogenization procedure developed here gives accurate expressions for the leading order (averaged) solution as well as higher order corrections. Two types of periodic permeability functions are considered here: smoothly and continuously varying and piecewise constant variations over layers of finite thickness. A brief summary is given in Section 5.

2. Horizontal draining layers:

The problem under consideration is the one-dimensional gravitational draining of a heterogeneous and fully-saturated porous medium with intrinsic permeability k that varies rapidly in the vertical direction. A sketch of this configuration is given in Fig. 1. We assume that the liquid volume fraction of the drainable pore space, f_ℓ, the fluid viscosity, μ, and the fluid density, ρ, are each constant. Mathematically, this problem can be expressed as

$$\frac{\partial}{\partial z}\left(K(Z)\frac{\partial \phi}{\partial z}\right) = 0, \tag{1}$$

$$\phi = 0, \qquad \text{at } z = 0, \tag{2}$$

$$\phi = h, \qquad \text{at } z = h, \tag{3}$$

$$\frac{dh}{dt} = -K(h/\epsilon)\frac{\partial \phi}{\partial z} \qquad \text{at } z = h, \tag{4}$$

subject to the initial profile $h(t = 0) = h_I$. The field variable $\phi(z, t)$ is related to the pressure p, gravity g and fluid density by $\phi = p/(\rho g)+z$ and $h(t)$ is the unknown position of the free surface. The quantity $K = k\rho g/(f_\ell \mu)$ is the hydraulic conductivity divided