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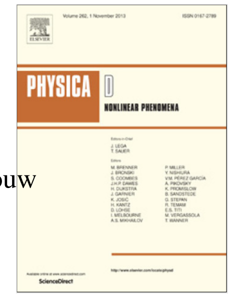

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ACCEPTED MANUSCRIPT

Synchronization of impacting mechanical systems with a single constraint

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Abstract

This paper addresses the synchronization problem of mechanical systems subjected to a single geometric unilateral constraint. The impacts of the individual systems, induced by the unilateral constraint, generally do not coincide even if the solutions are arbitrarily 'close' to each other. The mismatch in the impact time instants demands a careful choice of the distance function to allow for an intuitively correct comparison of the discontinuous solutions resulting from the impacts. We propose a distance function induced by the quotient metric, which is based on an equivalence relation using the impact map. The distance function obtained in this way is continuous in time when evaluated along jumping solutions.

The property of maximal monotonicity, which is fulfilled by most commonly used impact laws, is used to significantly reduce the complexity of the distance function. Based on the simplified distance function, a Lyapunov function is constructed to investigate the synchronization problem for two identical one-dimensional mechanical systems. Sufficient conditions for the uncoupled individual systems are provided under which local synchronization is guaranteed. Furthermore, we present an interaction law which ensures global synchronization, also in the presence of grazing trajectories and accumulation points (Zeno behavior). The results are illustrated using numerical examples of a 1-DOF mechanical impact oscillator which serves as stepping stone in the direction of more general systems.

Keywords: Synchronization, measure differential inclusion, unilateral constraint, Lyapunov stability, hybrid system

1. Introduction

Synchronization of coupled dynamical systems leads to motion in unison which is a fundamental phenomenon appearing in, for example, biological and engineering systems. The synchronization of chaotic oscillators, neural systems and mechanical systems has been studied extensively, see [1–5] and references therein. For both diffusively coupled differential equations and impulsively coupled maps, synchronization properties are generally studied through the analysis of the error dynamics which describes the difference between the states of the systems [1, 3, 6]. The error dynamics is typically characterized by a smooth differential equation or map and, consequently, linearization techniques and bifurcation theory have allowed to describe the convergence properties of the error dynamics and to study the effect of the interaction network. In this manner, the effect of the network topology, coupling strengths and delays of the network interaction on (master-slave, partial or global) synchronization is relatively well-understood for smooth systems [4, 7, 8] as well as for phase-coupled oscillators, which are naturally analyzed using Poincaré sections [9]. In contrast, synchronization of nonsmooth systems has received significantly less attention and, to the best of the authors' knowledge, the problem of synchronization for unilaterally constrained mechanical systems has not yet been addressed.

In this paper, synchronization is analyzed for mechanical systems with a single geometric unilateral constraint, which occur generally if mechanical systems (such as, e.g., robots) interact with a rigid environment. The dynamics of these systems comprises impacts which induce velocity jumps, rendering the system dynamics of an impulsive, hybrid nature [10–13]. Accumulation of infinitely many impacts in a finite time interval, which is known as Zeno-behavior, is a natural feature of unilaterally constrained mechanical systems. To describe the dynamics which includes such accumulation events, system models in terms of Measure Differential Inclusions (MDIs) are employed in [10, 14–16].

Because impacts of unilaterally constrained mechanical systems are a consequence of collisions and therefore are state-triggered events (i.e., occur at a certain position), they generally do not occur at the same time instants for nearby trajectories. Therefore, one expects a small time-mismatch of the impact time instants even for arbitrarily close initial conditions. A small Euclidean synchronization error prior to the first impact therefore does not imply that the Euclidean error is small during the time in between the impacts of two neighboring trajectories. As an illustrative example, Fig. 1 shows the time evolution of $V(t) = \|q_1 - q_2\|^2 + \|\dot{q}_1 - \dot{q}_2\|^2$ evaluated along the solutions of two synchronizing 1-DOF impacting systems with the general-

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