Modulation theory, dispersive shock waves and Gerald Beresford Whitham

A.A. Minzoni [a], Noel F. Smyth [b],*

[a] *Department of Mathematics and Mechanics, I.I.M.A.S., National Autonomous University of Mexico, 01000 Mexico, D.F., Mexico*
[b] *School of Mathematics, University of Edinburgh, Edinburgh EH9 3FD, Scotland, UK*

HIGHLIGHTS

- Short biography of Gerald Beresford Whitham.
- History of Whitham's development of modulation theory.
- Importance and applications of modulation theory emphasised.
- Development of dispersive shock wave theory outlined.
- Relation of modulation theory to Special Issue discussed.

ARTICLE INFO

ABSTRACT

Gerald Beresford (GB) Whitham, FRS, (13th December, 1927–26th January, 2014) was one of the leading applied mathematicians of the twentieth century whose work over forty years had a profound, formative impact on research on wave motion across a broad range of areas. Many of the ideas and techniques he developed have now become the standard tools used to analyse and understand wave motion, as the papers of this special issue of *Physica D* testify. Many of the techniques pioneered by GB Whitham have spread beyond wave propagation into other applied mathematics areas, such as reaction–diffusion, and even into theoretical physics and pure mathematics, in which Whitham modulation theory is an active area of research. GB Whitham's classic textbook *Linear and Nonlinear Waves*, published in 1974, is still the standard reference for the applied mathematics of wave motion. In honour of his scientific achievements, GB Whitham was elected a Fellow of the American Academy of Arts and Sciences in 1959 and a Fellow of the Royal Society in 1965. He was awarded the Norbert Wiener Prize for Applied Mathematics in 1980.

1. Academic career

Gerald Beresford (GB) Whitham (see Fig. 1) was born on the 13th December, 1927 in Halifax, North Yorkshire, in the United Kingdom and remained a proud Yorkshireman all his life. He was educated at Elland Grammar School, Elland, Yorkshire. GB then attended the University of Manchester in the neighbouring county of Lancashire, England, obtaining a B.Sc. honours degree in Mathematics in 1948, an M.Sc. degree in 1949 and a Ph.D. degree in 1953. His thesis advisor was Professor M.J. Lighthill, FRS, another great applied mathematician of the twentieth century. The period after the Second World War saw the development of the jet engine which enabled supersonic flight. There was then much mathematical interest in supersonic flow and shock waves, so that Lighthill suggested research in this area to GB, even though he initially wanted to work on general relativity. GB's thesis title was "Propagation of a spherical blast". Lighthill was later to state [1].

"I feel some pride at having, nineteen years ago, chosen as the problem to give to the first really good research student I had, the estimation of pressure pulses transmitted to great distances by bodies moving at supersonic speed."

GB met his wife, Nancy Whitham (née Lord), while he was a Ph.D. student at the University of Manchester. The story which GB told was that they met when GB was a tutor for a mathematics course which Nancy was taking. Nancy Whitham also obtained an honours B.Sc. degree in Mathematics at the University of Manchester. This background enabled her to help edit GB's classic textbook *Linear and Nonlinear Waves* [2]. Nancy Lord was born on the 29th December, 1929 in Oldham, Lancashire, United Kingdom,

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* Corresponding author.
E-mail address: N.Smyth@ed.ac.uk (N.F. Smyth).
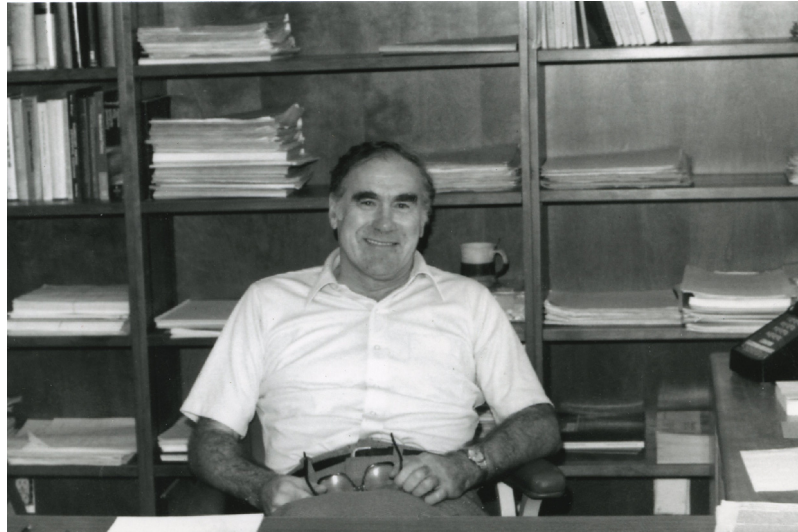
Fig. 1. GB Whitham in his office at the California Institute of Technology. Photograph courtesy of Sheila Shull of the California Institute of Technology.

the daughter of Frank and Amy Lord (née Buckley). She and GB married on the 1st September, 1951 in Manchester. They had three children, Ruth, Michael and Susan.

Richard Courant visited Manchester University in 1950. GB was invited by Courant to be a research associate at the Institute for Mathematics and Mechanics, now the Courant Institute of Mathematical Sciences, of New York University, where he was from 1951 to 1953. The work for GB's thesis was completed in 1951, but the actual degree was awarded in 1953 due to GB being in the United States at the Institute for Mathematics and Mechanics during the intervening period. GB felt that the emphasis at the Courant Institute was too pure for his tastes. He also felt that he should return to the United Kingdom. He then returned to the University of Manchester as a lecturer in applied mathematics in 1953, which position he held until 1956. However, he missed the atmosphere of the Courant Institute and the higher standard of living in the United States, so moved back to the Courant Institute as an associate professor in applied mathematics in 1956 and remained until 1959. The old feeling that at the Courant Institute there was too much of an emphasis on theorems and that there were few people who had physical type reasoning returned, so he became restless. Through the influence of George Carrier and Sydney Goldstein at Harvard University, the latter at the University of Manchester when GB was a student, and C.C. Lin at the Massachusetts Institute of Technology (MIT), Whitham moved to Boston to become a professor of mathematics at MIT, where he stayed from 1959 to 1962. The more applied bent of the mathematics there appealed to GB, but he found MIT to be very large and lacking in intimacy as compared to the Courant Institute.

Hans Liepmann of the Graduate Aeronautical Laboratories of the California Institute of Technology (GALCIT) visited Harvard University in the summer of 1960. He suggested that GB visit GALCIT and the California Institute of Technology (Caltech) as a visiting Professor of Applied Mechanics, with an eye to a future appointment. This GB did for a year from 1961 to 1962. He enjoyed Caltech immensely as it had the small, intimate size of the Courant Institute and the people with the physical applied mathematics orientation he could interact with. These included such famous names as Clark Millikan, H. Liepmann, L. Lees, P.A. Lagerstrom, J.D. Cole, D.E. Coles, F.E. Marble and A. Roshko. GB accepted an appointment as a professor of Aeronautics and Mathematics at Caltech in 1962, and so began his association with Caltech and the eventual Department of Applied Mathematics. He remained a professor of Aeronautics and Mathematics until

1967. Applied Mathematics at Caltech was then a part of the Department of Aeronautics. GB was the leading force in setting up a separate Department of Applied Mathematics at Caltech in 1967, becoming professor of applied mathematics from 1967 to 1983, serving as chairman of the Committee on Applied Mathematics from 1962 to 1971 and as the department's executive officer from 1971 to 1980. Under GB's guidance, the Department of Applied Mathematics at Caltech became one of the leading applied mathematics departments in the world. In 1983 he was appointed the Charles Lee Powell Professor, which position he retained until his retirement in 1998.

During his life, GB Whitham obtained scientific honours for his outstanding contributions to applied mathematics. The highest honour was being elected a Fellow of the Royal Society in 1965. GB was also elected a Fellow of the American Academy of Arts and Sciences in 1959. In 1980, he obtained one of the highest awards in applied mathematics, the Wiener Prize in Applied Mathematics. The description of this award states.

"The Wiener Prize is awarded for an outstanding contribution to "applied mathematics in the highest and broadest sense." Awarded jointly by the American Mathematical Society and the Society for Industrial and Applied Mathematics. This prize was established in 1967 in honour of Professor Norbert Wiener and was endowed by a fund from the Department of Mathematics of the Massachusetts Institute of Technology."

After his retirement, GB moved with his wife Nancy to Portland, Oregon to be nearer to their three children. He passed away there on the 26th January, 2014. For a complete obituary of GB Whitham, see the *Biographical Memoirs of Fellows of the Royal Society* [3]. On the 22nd August, 2015 a memorial bench dedicated to Professor Gerald B. and Mrs. Nancy Whitham and paid for by their family was unveiled outside the Firestone Laboratory at the California Institute of Technology (Fig. 2). The Firestone Laboratory was the location of the Department of Applied Mathematics (now the Department of Applied and Computational Mathematics) during the time that GB Whitham was at the California Institute of Technology.

2. Modulation theory

This special issue of *Physica D* on *Dispersive Hydrodynamics,* dedicated to GB Whitham, is especially appropriate as the year 2015 is the 50th anniversary of the publication of GB Whitham's seminal papers "Non-linear dispersive waves" [4] and "A general approach to linear and non-linear dispersive