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Traveling waves for a model of gravity-driven film flows in cylindrical domains

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HIGHLIGHTS

- Study a fully nonlinear dissipative/dispersive wave propagation model.
- Explore mean thickness threshold for traveling wave formation for viscous films.
- Compare threshold results to experiments.

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ABSTRACT

Traveling wave solutions are studied for a recently-derived model of a falling viscous film on the interior of a vertical rigid tube. By identifying a Hopf bifurcation and using numerical continuation software, families of non-trivial traveling wave solutions may be traced out in parameter space. These families all contain a single solution at a 'turnaround point' with larger film thickness than all others in the family. In an earlier paper, it was conjectured that this turnaround point may represent a critical thickness separating two distinct flow regimes observed in physical experiments as well as two distinct types of behavior in transient solutions to the model. Here, these hypotheses are verified over a range of parameter values using a combination of numerical and analytical techniques. The linear stability of these solutions is also discussed; both large- and small-amplitude solutions are shown to be unstable, though the instability mechanisms are different for each wave type. Specifically, for small-amplitude waves, the region of relatively flat film away from the localized wave crest is subject to the same instability that makes the trivial flat-film solution unstable; for large-amplitude waves, this mechanism is present but dwarfed by a much stronger tendency to relax to a regime close to that followed by small-amplitude waves.

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1. Introduction

The flow of falling liquid films is a topic of importance in several disciplines including biology and engineering. These films have a free surface whose evolution is governed by the interplay of body forces (gravity) and surface stresses (due to the surface tension of the free surface). Numerous modeling and experimental studies have advanced understanding of these flows in a variety of regimes corresponding to different parameter values (e.g., Reynolds number, Bond number, etc.), and in a number of geometrical setups including (i) along an inclined plane (e.g., [1,2]) and (ii) the exterior or interior of a vertical tube (e.g., [3–7]); see also [8]. The cylindrical geometry of the tube is distinct from the planar case due to the role of the free surface's azimuthal curvature in setting the surface stresses. This geometry is the focus of the current study, where we further concentrate on the cylinder *interior* problem. In contrast with its exterior film counterpart, this setup poses a natural limit to the thickness of the film, corresponding to cases when the surface tension azimuthal component drives the free surface all the way to the cylinder axis. When this happens, plugs of fluid that can fill sections of the tube are formed, up to the limit when the entire tube is filled with liquid moving according to the Poiseuille flow solution of the motion equations.

More specifically, the problem studied here is the gravity-driven downward flow of a highly viscous liquid film that coats the interior of a vertical rigid tube. While highly idealized, this particular setup is of interest due to its potential relevance for understanding the flow of the thin layer of mucus which lines

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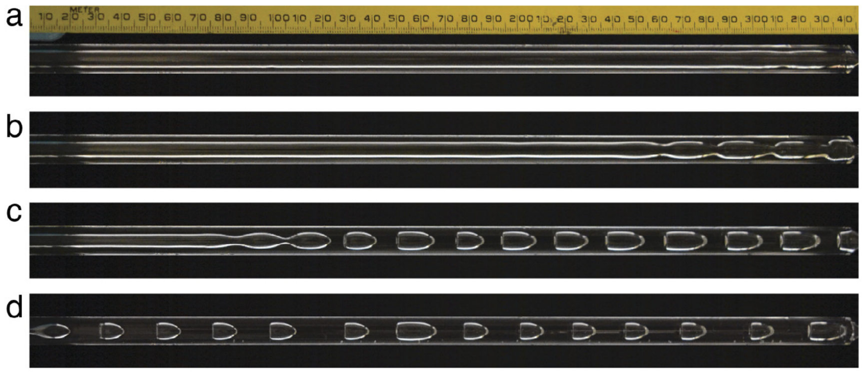

Fig. 1. Snapshots of four experiments with $\rho = 0.97$ g cm^{-3}, $\mu = 129$ P, $\gamma = 21.5$ dyn cm^{-1}, and $a = 0.5$ cm. The film thickness of each panel was measured to be (a) $h_0 = 0.223$ cm, (b) $h_0 = 0.256$ cm, (c) $h_0 = 0.295$ cm, and (d) $h_0 = 0.331$ cm. (Each snapshot is rotated by 90° with respect to the actual experiment, so that acceleration due to gravity acts from left to right.)
Source: Reproduced from [7].

human airways. The problem was studied experimentally in [7], where a fixed volume flux of a high-viscosity silicone oil was fed through an axisymmetric slit into the interior of a 40 cm long vertical tube. Once the entire tube was coated with oil, the free surface of the oil film was observed as it fell down the tube wall.

In these experiments, two distinct flow regimes were observed, distinguished from one another by a critical film thickness (as a function of other flow parameters). For relatively thick films, the free surface was observed to be unstable, with disturbances growing as they traveled down the tube until they 'pinched off' and formed liquid plugs clogging the tube; these plugs continued to travel downwards, eventually exiting the bottom of the tube. For thin films, the free surface either did not exhibit any observable instability growth or showed instabilities that remained small and did not clog the tube. In all observations, the film flow was observed to be axisymmetric; see Fig. 1.

Strongly nonlinear models for the axisymmetric flow studied here have typically been derived by assuming a small ratio of lengthscales and fall into one of two categories. Thin-film models rely on a small film thickness compared with the tube radius [3,9], while long-wave models utilize a small film thickness relative to a typical wavelength of free-surface disturbances [4,7]. See [10] for further discussion of this classification.

We next summarize a single-PDE long-wave model recently derived in [7]. The derivation of the model relies on both the aforementioned small long-wave aspect ratio and an assumed small Reynolds number so that inertia may be neglected. The dimensional form of the model is

$$\mu R_t = \rho g f_1(R; a) R_z + \frac{\gamma}{16R}[f_2(R; a)(R_z + R^2 R_{zzz})]_z, \tag{1}$$

where z is the independent axial coordinate and $R(z, t)$ denotes the position of the free surface. Here z is oriented so that gravity acts in the positive z direction; $r = 0$ denotes the center of the tube and $r = a$ denotes the tube wall. Experimental parameters include the fluid's molecular viscosity μ, density ρ, and surface tension γ; g is acceleration due to gravity. Subscripts will be used throughout the paper to denote partial derivatives, and the functions f_i are given by

$$f_1(R; a) = \frac{1}{2}[R^2 - a^2 - 2R^2 \log(R/a)], \tag{2a}$$

$$f_2(R; a) = -\frac{a^4}{R^2} + 4a^2 - 3R^2 + 4R^2 \log(R/a). \tag{2b}$$

The first term on the right-hand side (RHS) of (1) represents the effects of gravity; the remaining two terms represent the effects of surface tension acting through the azimuthal and axial curvatures of the free surface, respectively. The model (1) may also be expressed as a conservation law for the quantity R^2,

$$8\mu(R^2)_t = \{f_2(R; a)[-\rho g R^2 + \gamma(R_z + R^2 R_{zzz})]\}_z, \tag{3}$$

so that the model conserves the volume $\pi(a^2 - R^2)$ of the fluid film. This conservation of volume is one of the features distinguishing 'long-wave' models from most 'thin-film' models, which usually conserve an approximate volume $2\pi a(a - R)$ in this cylindrical geometry. For reference, the model equation is given in dimensionless form as well in the Appendix, however in what follows we will use the full dimensional form of the model equations for the most part.

For each mean film thickness $h_0 = a - R_0$, there is a trivial solution $R(z, t) = R_0$ to (1). This constant free surface is unstable to long-wave disturbances, consistent with linear stability results for the governing equations in this and related setups [11–13]; specifically, if a disturbance of the form $R = R_0 + A \exp[i(kz - \omega t)]$ is introduced to the model equation (1), the resulting dispersion relation is

$$\omega = -\frac{\rho g}{\mu} f_1(R_0; a) k + \frac{i\gamma}{16\mu R_0}\left[f_2(R_0; a)(-k^2 + R_0^2 k^4)\right]. \tag{4}$$

Thus the flat solution is unstable to a band of small wavenumbers $0 < k < R_0^{-1}$, with the fastest-growing wavenumber given by

$$k_m = (\sqrt{2}R_0)^{-1}. \tag{5}$$

The dispersion relation (4) has the same form as that of the well-studied Kuramoto–Sivashinsky (K–S) equation, first shown to be a limiting form of a model for film flow down an inclined plane [1,2] by Sivashinsky and Michelson [14].

When the model (1) is solved numerically using periodic boundary conditions and initial conditions consisting of a flat free surface perturbed by a superposition of several small-amplitude Fourier modes, the initial growth of the disturbances is well described by (4). As the perturbations grow beyond the weakly nonlinear regime, solutions exhibit one of two distinct types of behavior. For relatively thin films, the disturbances saturate as a series of traveling pulses which undergo various nonlinear interactions with one another, but generally hold their shape and propagate approximately as a coherent wave train. For thicker films, however, the fastest-growing disturbance continues to grow, apparently accelerated rather than saturated by nonlinearities in the model. This wave crest grows until its amplitude approaches the tube radius $r = 0$, with the latter stages of this growth occurring very rapidly. Due to the cylindrical geometry present in the model, as seen in the inverse powers and logarithms of R in (1), solutions cannot be computed once this crest reaches the tube

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