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Observation of dispersive shock waves developing from initial depressions in shallow water

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HIGHLIGHTS

- We observe dispersive shock waves in a shallow water tank.
- Different levels of nonlinearity and dispersion are contrasted.
- Data are compared with numerics based on the Korteweg–de Vries and Whitham equations.

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ABSTRACT

We investigate surface gravity waves in a shallow water tank, in the limit of long wavelengths. We report the observation of non-stationary dispersive shock waves rapidly expanding over a 90 m flume. They are excited by means of a wave maker that allows us to launch a controlled smooth (single well) depression with respect to the unperturbed surface of the still water, a case that contains no solitons. The dynamics of the shock waves are observed at different levels of nonlinearity equivalent to a different relative smallness of the dispersive effect. The observed undulatory behavior is found to be in good agreement with the dynamics described in terms of a Korteweg–de Vries equation with evolution in space, though in the most nonlinear cases the description turns out to be improved over the quasi linear trailing edge of the shock by modeling the evolution in terms of the integro-differential (nonlocal) Whitham equation.

1. Introduction

Dispersive shock waves (DSWs) are non-stationary wave trains that form spontaneously in weakly dispersive media [1]. The underlying mechanism is the wave steepening driven by the nonlinearity which leads to a gradient catastrophe, regularized by dispersion that becomes important close to the point where strong gradients are formed. Usually, the oscillations expand in a so-called shock fan characterized by a leading edge and a trailing edge, where the amplitude of the oscillations is largest and vanishingly small, respectively. DSWs constitute the dispersive counterpart of the viscous regularization of classical shock waves [2]; in the latter the dissipation dominates over dispersive effects.

Pioneering works on DSWs appeared between the 60's and the 70's. Sagdeev and coworkers predicted the oscillatory nature of the shock occurring in the extremely rarefied (collisionless) plasma [3]. The observation of such dispersive breaking in the lab was reported as early as 1970 [4]. In a seminal paper for the whole area of nonlinear waves, Zabusky and Kruskal [5] numerically investigated the evolution of a sine wave according to the weakly dispersive Korteweg–de Vries (KdV) equation [6–8], finding that the gradient catastrophe of the original waveform gives rise to oscillations which evolve into secondary waves with soliton features, eventually exhibiting recurrence of the input state after collisions [5]. Strictly speaking the wave packets emerging from the breaking of the periodic waves are multiple finite-gap solutions [9] which, however, resemble solitons, especially in the limit of weak dispersion where the Floquet bands dramatically shrink. However, DSWs can form also for initial conditions which possess no soliton content. A milestone towards a more general description was the solution of the Riemann problem (the evolution of a step initial datum) for the KdV, reported by Gurevich and

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Pitaevskii [10], who proposed the first explicit construction of the DSW by exploiting Whitham modulation theory [11]. The description of dispersive shocks still constitutes nowadays one of the most prolific application of such general averaging method proposed by Whitham.

The KdV has also played a pivotal role for the formulation of the limit of vanishing dispersion in the framework of the inverse scattering theory [12–16]. In particular the case of initial data with non-soliton content was addressed in [17,18]. Note that this limit is highly non-trivial since, at variance with the limit of vanishing viscosity, in dispersive settings it never leads to the classical (dispersionless) shock wave, since the oscillations become shorter and denser as the dispersion get weaker. Nowadays the KdV still remain a very important equation as it allows for testing more accurate asymptotic descriptions of the oscillatory zone [19–21]. However, it was realized since the beginning that DSWs constitute an ubiquitous behavior in several other dispersive Hamiltonian systems [22–25]. A remarkable universal example is the nonlinear Schrödinger (NLS) equation. In this context, experimental results on DSWs have been recently reported in the field of Bose–Einstein condensed atoms [26–31] and nonlinear optics [32–39]. Such experiments have also contributed to clearly highlight the contrast associated with solitonic-type of DSWs [26,36,39] (see also [40–43] for theoretical aspects), and nonsoliton DSWs (e.g., [27,32–34]). It should be noted that DSWs are also observed in nonintegrable systems, for which modulation equations can be still introduced, see for example [44].

In the context of water waves, DSWs (we stick to the term DSW for interdisciplinary purpose, though in the literature in this area, the term "undular bore" is more usually encountered) have also a long dating history. Important theoretical contributions came from Benjamin and Lighthill [45], Peregrine (who employed a model known as Benjamin–Bona–Mahony (BBM) equation [46]), and Johnson (who also investigated the effect of viscosity by means of a KdV–Burger model [47]). The most common situation is that of a bore moving into still water; for moderate amplitudes, it gives rise to undular behavior while, for larger amplitudes, undulations are still observed but the first wave is breaking. In the strongly nonlinear regime, no undulations are observed and a turbulent breaking front propagates. These phenomena can be observed in nature, with spectacular manifestations involving tidal bores in river estuaries (e.g., the Dordogne river in France, the Severn river in Wales, the Qiantang river in China, etc.), where the undular bores are also known under different local names [48].

Apparently, the laboratory investigations of undular bores was pioneered by Favre as early as 1935 [49]. Indeed, the secondary waves produced by the steep bore are also termed in hydraulic applications as Favre waves [50,51]. However, it is again in the seventies that laboratory experiments performed in shallow water with long waves have been reported and interpreted in terms of KdV dynamics [52–57]. Later review of such experiments have also pointed out the importance that the dynamics of the generated wavetrains can have in the interpretation of seismic generated tsunamis [58]. However, those experiments mainly dealt with initially positive elevations above the water surface, which produce multiple solitons. Only occasional observations were reported for smooth depressions, a case which cannot be interpreted in terms of generated solitons [53,55,56]. Moreover, such measurements suffered from limitations arising from the length of the wave-tank and by the technique used to launch the waves, employing a vertically moving piston. In this paper, we show that very extended and clean DSWs can be excited in a long tank (90 m) by using a wave maker which allows for a good degree of accuracy over the initial shape. In particular, we focus on initial depressions with profile close to square hyperbolic secant. In the initial stage where dispersion plays a negligible role, the wave evolves according to the Hopf (or inviscid Burger) equation, and experiences rarefaction on one edge and steepening over the opposite edge. The DSW that emerges from the steepened front, must be interpreted, in this case, as a genuine modulated nonlinear periodic function which is spontaneously generated due to the action of dispersion. We characterize the expansion of such DSW, comparing with numerical simulations based on a suitable form of the KdV equation and its extension introduced by Whitham. The regime that we investigate allows for observing a quite regular and extended oscillatory zone. Conversely, the length of the tank precludes the possibility to investigate the long-term asymptotic where one could expect major differences with the case of solitonic DSW (in the latter case, several solitons would asymptotically separate, as it would be the case for a positive square hyperbolic secant of proper amplitude). The characterization of the mid-term DSW developing from the depression is also useful in view of further studies devoted to study the interaction of genuine solitons and DSWs which can occur for more general initial shapes.

The paper is organized as follows. In Section 2 we present the asymptotic models that we employ in order to describe the experiment, emphasizing that such models are written in such a way to evolve time series in space. In Section 3 we present the experimentally observed data, and in Section 4 we discuss the numerical modeling of our observations. Finally, we summarize our finding in Section 5.

2. The Korteweg–de Vries equation and the Whitham equation in their spatial evolution form

The fully nonlinear viscous equations that describe the evolution of surface gravity waves are definitely too much complicated (even from a numerical treatment) to understand basic mechanisms such as solitons, breathers or DSWs. Therefore, approximations are needed if one is interested in capturing some specific nonlinear wave dynamics. Indeed, Boussinesq [7] and Korteweg and de Vries [6] made use of asymptotic methods for deriving what is now known as the KdV equation (for discussion on the differences between the methods used in the derivation see [8]). The motivation of their work was the physical explanation of the observation of the "Wave of Translation" made by Scott Russell in 1834.

The classical derivation of the KdV equation (see for example [2]) from the Navier–Stokes equations requires a number of hypotheses: the fluid is considered inviscid and the flow irrotational; waves have long wavelength and propagation in only one direction is allowed. The key point in the derivation is the introduction of two nondimensional parameters: the first one is the nonlinear parameter, $\alpha = \eta_0/h$, where η_0 is a characteristic wave amplitude and h is the unperturbed water depth; the second one is the dispersive or the shallow water parameter, $\beta = kh$, with k a characteristic wave number of the problem under examination. "Waves of Translations" with a permanent form are the result of a balance between nonlinearity and dispersion, therefore the KdV equation is obtained by balancing α and β. Note that if one expands the unidirectional dispersion relation for water waves, $\omega(k) = \sqrt{gk\tanh(kh)}$, in powers of kh, i.e. in the shallow water limit, at the leading order the dynamics turns out to be nondispersive, $\omega = \sqrt{gh}\,k$; therefore, if one is interested in balancing nonlinearity and dispersion, then one should choose $\alpha \sim \beta^2$. This is the fundamental assumption for the derivation of the KdV equation.

In dimensional variables the KdV equation takes the following form:

$$\eta_t + c_0\eta_z + \frac{3}{2}\frac{c_0}{h}\eta\eta_z + \frac{1}{6}c_0h^2\eta_{zzz} = 0 \tag{2.1}$$

where $c_0 = \sqrt{gh}$ is the phase velocity of linear waves and z the propagation coordinate. When dealing with experimental data