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Sine–Gordon modulation solutions: Application to macroscopic non-lubricant friction

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HIGHLIGHTS

- Theoretical model of dry macroscopic friction has been developed.
- Model is based on the sine–Gordon modulation (Whitham) equations.
- Model connects the kinetic and dynamic parameters of the frictional process.
- Model describes seismic events in wide range of rupture and slip velocities.

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ABSTRACT

The Frenkel–Kontorova (FK) model and its continuum approximation, the sine–Gordon (SG) equation, are widely used to model a variety of important nonlinear physical systems. Many practical applications require the wave-train solution, which includes many solitons. In such cases, an important and relevant extension of these models applies Whitham's averaging procedure to the SG equation. The resulting SG modulation equations describe the behavior of important measurable system parameters that are the average of the small-scale solutions given by the SG equation.

A fundamental problem of modern physics that is the topic of this paper is the description of the transitional process from a static to a dynamic frictional regime. We have shown that the SG modulation equations are a suitable apparatus for describing this transition. The model provides relations between kinematic (rupture and slip velocities) and dynamic (shear and normal stresses) parameters of the transition process. A particular advantage of the model is its ability to describe frictional processes over a wide range of rupture and slip velocities covering seismic events ranging from regular earthquakes, with rupture velocities on the order of a few km/s, to slow slip events, with rupture velocities on the order of a few km/day.

1. Introduction

An understanding of tribology and its complicated nonlinear aspects requires a combination of experimental, theoretical and computational efforts [1,2]. While experiments and molecular dynamics simulations provide invaluable information about the atomic origins of static and dynamic friction, the complexities of realistic 3D systems make it difficult to understand the general mechanisms underlying friction. In this regard, simple low-dimensional phenomenological models, such as the Tomlinson [3] and Frenkel–Kontorova (FK) [4] models are useful tools for determining the essential features of nonlinear sliding phenomena [5,6]. These features can then be tested by experiment and molecular dynamics simulation. Thus, development of such models is an essential part of studying friction.

The relative movement of two solids in contact is accompanied by friction occurring due to interactions between surface asperities. Under static conditions or sliding at uniform velocity, friction is usually described by the frictional coefficient, *i.e.,* the proportionality coefficient between tangential and normal stress (classical Amontons–Coulomb law). However, this description is not sufficient for sliding at non-uniform velocity. Multiple experiments have shown that in transitional regimes, friction depends on slip, sliding rate, contact time and normal stress history (see the extensive reviews [7–9]). Over the past 50 years, various approaches for the modeling of non-uniform frictional processes have been

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developed. Two types of models are the most common, i.e., mass–spring models [10–17], and rate-and-state (Dietrich–Ruina) models [18–28].

The FK model (one of the mass–spring models) has been widely used to study micro/nanoscopic friction [29–39]. We demonstrated that this model may also describe the dynamics of macroscopic non-lubricant friction [40–42]. In the model we proposed, sliding is analogous to plasticity. It occurs due to *movement of a certain type of defect* (a "macroscopic dislocation") nucleated on the frictional surfaces by shear stress in the presence of asperities. The movement of dislocations (areas on the frictional surfaces with accumulated stress) requires much less external shear force than uniform displacement of frictional surfaces. The advantages of this model are: (1) it is an intrinsically dynamical model, rooted in the Newtonian equations of motions; (2) parameters used in the model have explicit and unambiguous physical correlates; (3) it describes frictional processes over a wide range of conditions, from very fast processes such as regular earthquakes down to very slow processes such as creep, silent, and slow earthquakes [40–45].

In the continuum limit the FK model is described by the sine–Gordon (SG) equation, one of the fundamental universal nonlinear equations of mathematical physics. Among other applications, this equation has been used in the theory of dislocations, Josephson junctions, self-induced transparency, commensurate–incommensurate phase transitions, charge-density waves, magnetic domain walls, etc. (see [46–49] and references therein). Due to its universal character, the SG equation has been extensively investigated [46,50–52]. The mathematical apparatus which has been developed is fully applicable to the problems considered here. What distinguishes our approach compared to other mass–spring frictional models, in general, and FK models, in particular, is the use of the SG modulation equations rather than the standard SG equation itself. Solutions to the latter provide detailed descriptions of system parameters at small scales that are typically unmeasurable. The advantage of our approach is that it describes the average behavior of parameters, such as slip and rupture velocities and shear stress distribution, which are measurable parameters of interest.

The organization of the paper is as follows. We first describe the basics of the model, followed by the solution of the modulation equations. Applications are then considered. The paper concludes with a summary of our results.

2. Model description

2.1. Model derivation

It is commonly accepted that macroscopic friction results from the interaction between asperities. During relative movement of the frictional surfaces, an asperity on one frictional surface detaches from an asperity on the opposite frictional surface and attaches to the next opposing asperity, and this process continues as long as the frictional surfaces are sliding relative to each other (see illustration in Fig. 1). Neighboring asperities on the same surface interact with each other as part of an elastic solid. We will consider the asperities (Fig. 1(b)) on one of the frictional surfaces as forming a linear chain of balls of mass M, each ball interacting with the nearest neighbors on either side via spring forces of constant K_b (Fig. 1(c)). The asperities on the opposite frictional surface will be regarded as forming a rigid substrate which interacts with the masses M via a periodic potential. Then we can apply the one-dimensional FK model to describe the slip dynamics:

$$M\frac{\partial^2 u_i}{\partial t^2} - K_b(u_{i+1} - 2u_i + u_{i-1}) + F_d \sin\frac{2\pi}{b}u_i = F(x,t) - f_i\left(x, t, \frac{\partial u_i}{\partial t}\right), \quad (1)$$

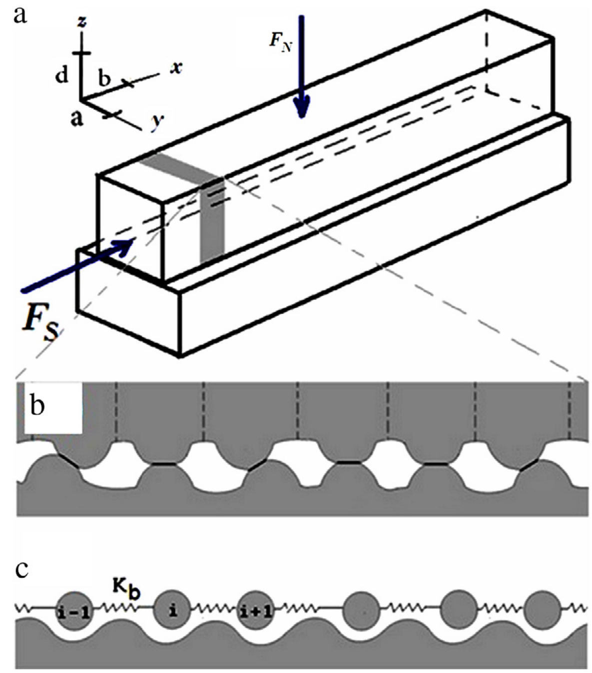

Fig. 1. Schematic of (a) experimental arrangement, (b) asperity contact, and (c) chain of masses interacting via elastic springs and placed in a periodic potential (substrate). The balls represent asperities. The sine-shaped surface is the opposite plate.

where u_i is the shift of ball i relative to its equilibrium position, b is a typical distance between asperities, t is time, F_d is the amplitude of the periodic force on M associated with the periodic substrate potential, f_i is the frictional (dissipative) force on asperity i, and F is the external force. This equation has been used to study plasticity in crystalline materials, which involves the dynamics of atomic-scale edge dislocations [46,53,54]. To express the coefficients of Eq. (1) in terms of the volume and surface mechanical parameters of the frictional blocks and external conditions such as normal stress, we first consider these coefficients for the case of plasticity, i.e., at the atomic scale. We assume a sliding surface parallel to the actual frictional surface but inside the block. Supposing that it is a crystal material with volume density ρ and interatomic distances a, b and d in the directions shown on Fig. 1(a), we can find the coefficients for Eq. (1) (see [46,53,54] for details): $M = \rho abd$, $K_b = \frac{2\mu ad}{(1-\nu)b}$, $F_d = \frac{\mu b^2 a}{2\pi d}$, where μ is the shear modulus and ν is the Poisson ratio. Now Eq. (1) can be written in the form (the second term on the left hand side is obtained in the continuum limit approximation):

$$\frac{c^2\rho ad}{2\pi}\frac{\partial^2(2\pi u/b)}{\partial(tc/b)^2} - \frac{2\mu ad}{2\pi(1-\nu)}\frac{\partial^2(2\pi u/b)}{\partial(x/b)^2} + \frac{\mu b^2 a}{2\pi d}\sin\left(\frac{2\pi u}{b}\right) = F - f, \quad (2a)$$

where $c^2 = 2\mu/(\rho(1-\nu)) \equiv c_l^2(1-2\nu)/(1-\nu)^2$ and c_l is the longitudinal acoustic velocity. Note that $c_s < c < c_l$, where c_s is the shear wave velocity. An equivalent form is (supposing for simplicity that $a = b = d$):

$$\frac{\partial^2(2\pi u/b)}{\partial(tc/b)^2} - \frac{\partial^2(2\pi u/b)}{\partial(x/b)^2} + A^2\sin\left(\frac{2\pi u}{b}\right) = (F-f)\frac{2\pi A^2}{\mu b^2}. \quad (2b)$$

The dimensionless parameter A is as $A = ((1-\nu)/2)^{1/2}$. In the derivation of Eq. (2b), A^2 is essentially the ratio of the amplitude of two forces: one is the force amplitude between an atom and the substrate layer and the other is the force