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Incoherent shock waves in long-range optical turbulence

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ABSTRACT

Considering the nonlinear Schrödinger (NLS) equation as a representative model, we report a unified presentation of different forms of incoherent shock waves that emerge in the long-range interaction regime of a turbulent optical wave system. These incoherent singularities can develop either in the temporal domain through a highly noninstantaneous nonlinear response, or in the spatial domain through a highly nonlocal nonlinearity. In the temporal domain, genuine dispersive shock waves (DSW) develop in the spectral dynamics of the random waves, despite the fact that the causality condition inherent to the response function breaks the Hamiltonian structure of the NLS equation. Such spectral incoherent DSWs are described in detail by a family of singular integro-differential kinetic equations, e.g. Benjamin–Ono equation, which are derived from a nonequilibrium kinetic formulation based on the weak Langmuir turbulence equation. In the spatial domain, the system is shown to exhibit a large scale global collective behavior, so that it is the fluctuating field as a whole that develops a singularity, which is inherently an incoherent object made of random waves. Despite the Hamiltonian structure of the NLS equation, the regularization of such a collective incoherent shock does not require the formation of a DSW — the regularization is shown to occur by means of a different process of coherence degradation at the shock point. We show that the collective incoherent shock is responsible for an original mechanism of spontaneous nucleation of a phase-space hole in the spectrogram dynamics. The robustness of such a phase-space hole is interpreted in the light of incoherent dark soliton states, whose different exact solutions are derived in the framework of the long-range Vlasov formalism.

1. Introduction

Shock waves have been thoroughly investigated during the last century in many different branches of physics [1]. The well-known phenomenon of viscous shock wave in a dissipative compressible fluid (gas) is characterized by a steep jump in gas velocity, density, and temperature across which dissipation of energy due to particle collisions regularizes the shock singularity. On the other hand, in conservative systems a different regularization occurs that entails the formation, owing to dispersion, of rapidly oscillating non-stationary structures, so-called undular bores or dispersive shock waves (DSWs). Their theoretical study was pioneered in plasma physics [2,3] and water waves [4], and was readily followed by lab observations [5,6]. Seminal contributions arose afterward in the context of the celebrated integrable Korteweg–De Vries (KdV) equation, both in terms of construction of non-stationary DSWs [7] based on Whitham modulation theory [8] and a formulation of the weak dispersion limit based on inverse scattering [9]. However, it became soon clear that DSW phenomena constitute a universal signature of singular nonlinear wave behavior in Hamiltonian models, regardless of the property of integrability [10,11]. Such behavior has generated continued interest among diverse areas of physics, ranging from the interpretation of natural phenomena such as atmospheric gravity waves [12], oceanic internal waves[13], or tidal bores [14], to lab experiments in Bose–Einstein condensates [15], unitary Fermi gases [16], nonlinear optics (temporal [17], and spatial [18] phenomena, as well as a diversity of optical settings [19]), quantum liquids [20], nonlinear chains or granular materials [21], viscous fluids [22], and electron beams [23]. Also notice that the role of *structural disorder of the medium* on the properties of DSWs has been investigated in the context of optical waves [24,25].

These previous studies on DSWs have been essentially reported for coherent, i.e., deterministic, wave envelopes. When studying,

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vice versa, a system of fully random nonlinear waves (a speckle beam in the language of optics), the usual dynamics of DSW formation is challenged, yet the formation of incoherent shocks becomes possible through different mechanisms, as we have shown in two recent works [26,27]. In this respect, it is important to remind that an accurate statistical description of a system of random waves has been developed in the weakly nonlinear regime by the so-called wave turbulence theory, which has been successfully applied to a huge variety of physical systems [28–35]. However, such an approach is known to break down for strong nonlinearities, when the turbulent system can be heavily affected by nonlinear excitations, such as shock waves, vortices, (quasi-)solitons, collapsing wavepackets, or rogue waves [30–32,35–42]. In this general framework, we recently explored how shock wave singular behaviors can spontaneously emerge within two particular types of turbulent systems which are frequently encountered in the context of optical waves.

(i) On the one hand, we considered the temporal dynamics of a random wave that propagates in a defocusing nonlinear medium characterized by a temporal noninstantaneous nonlinear response (i.e., temporal nonlocality). In this case, at variance with the deterministic case where breaking occurs in time domain [17], the field retains a random structure in time, while exhibiting a wave breaking process ("gradient catastrophe") in frequency which leads to incoherent DSWs in the Fourier spectral dynamics [26,43]. On the basis of a weakly nonlinear wave turbulence approach, the spectral dynamics of the incoherent wave can be described in the framework of a nonequilibrium kinetic equation whose structure is formally analogous to that considered to study weak Langmuir turbulence (WLT) in plasmas [34,35]. Note that this formalism proved efficient in describing different optical phenomena [35], such as the formation of spectral incoherent solitons [44,45] through supercontinuum generation [46]. In Ref. [26] we showed that spectral incoherent DSWs can be described in detail by a family of singular integro-differential kinetic equations (SID-KE), which were derived from the WLT kinetic equation in the limit of a long-range nonlinear interaction, i.e., a highly noninstantaneous response of the nonlinearity. This approach revealed interesting links with the 3D vorticity equation in incompressible fluids [47], or the integrable Benjamin–Ono (BO) equation [48] originally derived in hydrodynamics for stratified fluids and recently investigated in the semi-classical limit to study coherent wave breaking processes [49].

(ii) On the other hand, we considered the (transverse) spatial dynamics of a random wave that propagates in a nonlinear medium characterized by a highly nonlocal nonlinear response, i.e., spatial long-range interaction. A wave turbulence approach of the problem revealed that this regime is described in detail by a nonequilibrium long-range Vlasov formalism [50]. Note that this kinetic formulation differs from the traditional Vlasov equation describing random waves in hydrodynamics [37,40,41], in plasmas [51], or in optics, such as e.g., incoherent modulational instabilities [35,52], or incoherent solitons [52–54], while its structure is formally analogous to that describing systems of particles with long-range, e.g., gravitational, interactions [55,56]. In a recent work [27], we reported both theoretically and experimentally, a characteristic transition in the turbulent system: By strengthening the nonlocal character of the nonlinear response, the system evolves from a fully turbulent regime featuring a sea of coherent small-scale dispersive shock-waves ('shocklets') toward the unexpected emergence of a giant collective incoherent shock wave. The originality of this latter phenomenon of collective shock stems from the fact that, as a result of the underlying long-range interaction, the system exhibits a global collective behavior, in the sense that it is the random wave as a whole which leads to the formation of a shock wave: The shock singularity is inherently an incoherent object itself made of random waves. As a consequence of this collective behavior, the regularization of the incoherent shock does not require the formation of a DSW structure — the regularization occurs by means of a mechanism of coherence degradation that occurs at the shock front [27].

Considering the nonlinear Schrödinger (NLS) equation as a representative model, we provide in this article a unified presentation of these two different forms of incoherent shock singularities that develop in the long range interaction regime of the turbulent system. In Section 2 we give a brief overview on spectral incoherent DSWs, in particular by underlying the essential properties which distinguish them from the collective incoherent shock waves discussed in Section 3. In this respect, we remark that both cases challenge the usual scheme underlying the DSW formation in Hamiltonian systems, since (i) in the temporal case, genuine oscillatory DSW structures are formed (though in Fourier space), in spite of the fact that the model equation is non-Hamiltonian due to the causality constraint, whereas (ii) the usual deterministic DSW regularization is 'inhibited' in the spatial case, in spite of the Hamiltonian structure of the spatial NLS equation. An other remarkable difference is that the development of collective incoherent shocks requires, as usual, a strong nonlinear interaction, whereas spectral incoherent DSWs are generated in the weakly nonlinear turbulent regime. In Section 3 we provide further physical insight into the nature of the collective incoherent shock wave recently observed in Ref. [27]. At variance with [27], we consider here the dynamics of a random nonlinear wave characterized by a hole in its envelope profile. Despite the underlying Hamiltonian structure of the system, such a hole perturbation usually exhibits a damping during the evolution, so that the system irreversibly relaxes toward an unperturbed homogeneous state as a result of an effective Landau-damping effect. However, in the strong nonlinear regime, we show that the system exhibits an incoherent shock singularity for the momentum and a collapse singularity for the intensity envelope of the random wave. The numerical simulations reveal that the regularization of such a double shock-collapse singularity is responsible, after a complex transient process, for the nucleation of a peculiar spectrogram hole in phase-space. This phase-space hole collective structure proves extremely robust in the system evolution, a property which we interpret in the light of incoherent dark soliton solutions that we derive from the long-range Vlasov equation. The analysis reveals that such incoherent dark soliton states cannot be clearly identified through the usual intensity analysis in real space, while their very nature appears to be 'hidden' in the phase-space representation.

2. Temporal domain: Spectral incoherent DSWs

2.1. Temporal nonlocal NLS equation

In this section we provide a brief overview on the nature of spectral incoherent DSWs which develop in the spectral dynamics of a random wave that evolves in a noninstantaneous nonlinear environment. The starting point is the temporal version of the NLS equation accounting for a delayed nonlinear response:

$$i\partial_z\psi = -s\partial_{tt}\psi + \psi\int R(t-t')\,|\psi|^2(t')\,dt'. \tag{1}$$

As usual in optics, the propagation distance z plays the role of an evolution 'time' variable, while the time (t) plays the role of the spatial variable [57]. The response function $R(t)$ is constrained by the causality condition, $R(t) = 0$ for $t < 0$, and the typical width of $R(t)$ denotes the nonlinear response time, τ_R. The problem has been normalized with respect to the 'healing time' $\tau_0 =$