Physica D 333 (2016) 323–335

Contents lists available at ScienceDirect

Physica D

journal homepage: www.elsevier.com/locate/physd

Nonlinear random optical waves: Integrable turbulence, rogue waves and intermittency

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HIGHLIGHTS

- Nonlinear propagation of random waves is studied in integrable systems.
- Statistical properties are examined in focusing and defocusing propagation regimes.
- Heavy and low-tailed deviations from Gaussian statistics are observed.
- Intermittency phenomenon is found in integrable turbulence.

ARTICLE INFO

Article history:
Received 2 September 2015
Received in revised form
25 March 2016
Accepted 1 April 2016
Available online 8 April 2016

Keywords:
Integrable turbulence
Nonlinear Schrödinger equation
Random waves
Statistics
Rogue waves
Intermittency

ABSTRACT

We examine the general question of statistical changes experienced by ensembles of nonlinear random waves propagating in systems ruled by integrable equations. In our study that enters within the framework of integrable turbulence, we specifically focus on optical fiber systems accurately described by the integrable one-dimensional nonlinear Schrödinger equation. We consider random complex fields having a Gaussian statistics and an infinite extension at initial stage. We use numerical simulations with periodic boundary conditions and optical fiber experiments to investigate spectral and statistical changes experienced by nonlinear waves in focusing and in defocusing propagation regimes. As a result of nonlinear propagation, the power spectrum of the random wave broadens and takes exponential wings both in focusing and in defocusing regimes. Heavy-tailed deviations from Gaussian statistics are observed in focusing regime while low-tailed deviations from Gaussian statistics are observed in defocusing regime. After some transient evolution, the wave system is found to exhibit a statistically stationary state in which neither the probability density function of the wave field nor the spectrum changes with the evolution variable. Separating fluctuations of small scale from fluctuations of large scale both in focusing and defocusing regimes, we reveal the phenomenon of intermittency; i.e., small scales are characterized by large heavy-tailed deviations from Gaussian statistics, while the large ones are almost Gaussian.

1. Introduction

The field of modern nonlinear physics has started with the pioneering work of Fermi and collaborators [1] who studied a chain of coupled anharmonic oscillators, now known as the Fermi–Pasta–Ulam (FPU) system, with the aim of understanding the effect of nonlinearities in the process of thermalization. Their unexpected results, i.e. the observation of a recurrent behavior instead of the phenomenon of thermalization, triggered the work by Zabusky and Kruskal [2] who performed numerical simulations of the Korteweg–de Vries equation (KdV), i.e. the long wave approximation of the FPU system, and made the fundamental discovery of solitons; such discovery lead to the development of a new field in mathematical physics that deals with integrable systems with an infinite number of degrees of freedom. Some years after the discovery, Zakharov and Shabat [3] found that the nonlinear Schrödinger equation (1D-NLSE) is an integrable partial differential equations and has multi-soliton just like the KdV equation. The following years were characterized by the search of new integrable equations and the study of their mathematical properties and solutions.

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http://dx.doi.org/10.1016/j.physd.2016.04.001

In the late seventies and early eighties, besides solitons on a zero background, a new class of solutions of the 1D-NLSE were found [4–7] that describe the instability of a finite coherent background. Those solutions are sometimes named "breathers": the classical Akhmediev [7] and the Peregrine [6] solutions breath just once in their life and they describe *in toto* the modulational instability process (also in its nonlinear stages). The Kuznetsov–Ma solution [4,5] is periodic in the evolution variable and the perturbation of the coherent state is never small.

The phenomenon of rogue waves (RW) in the ocean has been known to humans much before the discovery of integrability of partial differential equations; however, only in the last fifteen years a connection between those two fields has been made and it has been conjectured that the "breather" solutions of the focusing 1D-NLSE could be considered as rogue wave prototypes [8,9]. This idea has been rapidly picked up in different scientific communities [10–16] and a new research activity has started. A first important step was the reproduction of the breather solutions of the 1D-NLSE in water wave tanks [17,18] and in optical fibers [19–21]. In order to generate these coherent structures in controlled lab experiments, very specific and carefully-designed *coherent* initial conditions have been considered. However, in nature such conditions are almost never encountered; wind generates ocean waves *via* a non trivial mechanism [22,23] and the resulting wave field appears as a superposition of random waves characterized by Fourier spectra with a small, but finite, spectral bandwidth. This means that the problem of rogue waves must be investigated from a statistical point of view [24,25,15,13]. Indeed, one of the major questions to be answered in the field of rogue waves concerns the determination of the probability density function (PDF) of the wave field for some given initial and boundary conditions. This is definitely not an easy task and, nowadays, given a nonlinear partial differential equation, there is no systematic theory that allows one to determine the PDF of the wave field.

The field of rogue waves has "belong" to oceanographers until the pioneering experiments with optical fibers described in [26]. Since then, optical rogue waves have been studied in various contexts such as supercontinuum generation in fibers [26–30,14], propagation in optical fiber described by the "pure" 1D-NLSE [13] or with higher order dispersion [31,32], laser filamentation [33], passive cavities [34,32], lasers [35–38], photorefractive ferroelectrics [39] and Raman fiber amplifiers [40].

From the general point of view and beyond the question of rogue waves, the field of incoherent nonlinear optics has recently become a blooming area of research as illustrated by several important works about incoherent dark solitons [41], the modulational instability of incoherent waves [42–44], incoherent solitons in nonlocal nonlinear media [45,46], or spectral incoherent solitons in optical fibers [47]. More specifically, nonlinear fiber optics has recently grown as a favorable laboratory to investigate both statistical properties of nonlinear random waves and hydrodynamic-like phenomena [14,48–54]. Indeed, the field of incoherent dispersive waves resemble very much the classical field of fluid turbulence where, instead of waves, eddies interact with each other, giving birth to new eddies of different size. This mechanism is at the origin of the celebrated Kolmogorov cascade of the three-dimensional turbulence which is characterized by a constant flux of energy within the so called inertial range. A source and a sink of energy are required in order to maintain the cascade. Many years after such concept was developed, it was found that the cascade is intermittent, i.e. the statistical properties of the velocity field vary with the scales, becoming less normal for smaller scales, see for example [55] for references. In the light of the paper [56], this idea will be discussed in the present paper in the context of the dynamics of incoherent waves ruled by the integrable 1D-NLSE. Such equation provides a bridge between nonlinear optics and hydrodynamics, see [57] for a one to one comparison. In particular, the focusing 1D-NLSE describes at leading order the physics of deep-water wave trains and it plays a central role in the study of rogue waves [58,59,15,60,14]. Moreover, the focusing 1D-NLSE is the simplest partial differential equation that describes the modulational instability phenomenon that is believed to be a fundamental mechanism for the formation of RW [24,15].

As mentioned, such waves emerge in the ocean from the interplay of incoherent waves in turbulent systems. The theoretical framework combining a statistical approach of random waves together with the property of integrability of the 1D-NLSE is known as *integrable turbulence*. This emerging field of research recently introduced by V. Zakharov relies on the analysis of complex phenomena found in nonlinear random waves systems described by an integrable equation [61–64,56,65]. Strictly speaking, the word "turbulence" is not fully appropriate in the sense that the dynamics in Fourier space is not characterized by a constant flux of a conserved quantity because the system is Hamiltonian (no forcing and dissipation are included). For these integrable systems, given an initial condition, the spectrum generally relaxes to a statistically stationary state that in general is different from the standard thermal equilibrium characterized by the equipartition of energy. The prediction of the spectra of such final state and its statistical properties is the objective of the integrable turbulence field. In the weakly nonlinear regime [66], starting with *incoherent* initial conditions in the 1D-NLSE, deviation from Gaussian statistics has been predicted. In hydrodynamical numerical simulations performed with envelope equations and experiments made in water tanks, non Gaussian statistics of the wave height has also been found to emerge from random initial conditions [67,24,68].

While in the water wave context the NLSE is only a crude (but reasonable) approximation of the original equations of motion, the field of nonlinear fiber optics is a promising field for the investigation of integrable turbulence because optical tabletop "model experiments" accurately described by the 1D-NLSE can be performed [19–21,56]. Despite the numerous works devoted to optical RW, the generation of extreme events from purely stochastic initial conditions in focusing 1D-NLSE model experiments remains a crucial and open question [12,60,64,14].

In this paper, we review and extend a number of results recently obtained by the authors of this paper from optical fiber experiments [56,13] in the anomalous and normal dispersion regime. The dynamics of the waves in the considered fiber is described with high accuracy by the focusing and defocusing 1D-NLSE. In the focusing regime, the idea is to implement optical fiber experiments conceptually analogous to the water tank experiment described in [24] where waves with a finite spectral bandwidth and random phases are generated at one end of the tank and the evolution of the statistical properties of the wave field is followed along the flume. Using an original setup to overcome bandwidth limitations of usual detectors, we evidence strong distortions of the statistics of nonlinear random light characterizing the occurrence of optical rogue waves in integrable turbulence.

In the defocusing regime, modulational instability is not possible and the evolution of incoherent waves does not lead to the formation of rogue waves. The statistics of wave intensity, initially following the central limit theorem, changes along the fiber resulting in a decrease of the tails of the PDF. This implies that the probability of finding a rogue wave is lower than the one described by linear theory. Implementing an optical filtering technique, we also report on the statistics of intensity of light fluctuations on different scales and we observe that the PDF of the wave intensity shows tails that strongly depend on the scales. This reveals the phenomenon of intermittency, previously mentioned,