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Determination of the scattered fields of an SH-wave by an eccentric coating-fiber ensemble using DEIM

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ABSTRACT

To date, the existing theories pertinent to the determination of the scattered fields of an inhomogeneity have been limited to certain topological symmetries for which the method of wave-function expansion is widely used. In the literature the wave-function expansion method has also been employed to the case involving concentric coated fiber. An alternative approach is the dynamic equivalent inclusion method (DEIM) proposed by Fu and Mura [L.S. Fu, T. Mura, The determination of elastodynamic fields of an ellipsoidal inhomogeneity. ASME J. Appl. Mech. 50 (1983) 390–396.] who found the scattered field of a single spheroidal inhomogeneity. The pioneering work of Eshelby [J.D. Eshelby, The determination of the elastic field of an ellipsoidal inclusion, and related problems, Proc. R. Soc. London, Ser. A A241 (1957) 376–396.] on elastostatic EIM is based on polynomial form of eigenstrains which holds certain useful properties and subsequently its application is only effective for certain relevant situations and not necessarily efficient for other problems. Nevertheless, Fu and Mura's analysis is also based on polynomial eigenstrains. It will be shown that taking the dynamic homogenizing eigenstrains in the form of the series expansion whose general term is products of functions of r and trigonometric functions of θ, is more rigorous and attractive for the problem under consideration. This natural form of solution gives very accurate result with just the first few terms of the series. Moreover, this work aims to extend the DEIM to the case of coated fiber obstacle with the rather complex topology where the coating-fiber phases are not concentric. The effect of variableness of the coating thickness on the elastodynamic fields is examined. Comparison with other analytical solutions, whenever available, establishes the remarkable accuracy and robustness of the proposed theory.

1. Introduction

In this work an inhomogeneity is referred to a sub-domain of a medium whose elastic properties differ from its surrounding matrix, whereas an inclusion is a sub-domain having the same elastic properties as those of matrix, but possessing an eigenstrain field. A multi-inhomogeneity system is referred to a sequence of nested inhomogeneities embedded in an infinite body.

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Elastic fields due to a dynamically transforming spherical inclusion have been studied by Mikata and Nemat-Nasser [3]. They obtained the disturbance strain field within the inclusion due to a constant eigenstrain field and gave the corresponding dynamic Eshelby's tensor for the interior and exterior points. Later, Mikata and Nemat-Nasser [4] utilizing the treatment of Pao and Mow [5], addressed the problem of a dynamically transforming spherical inhomogeneity with uniform eigenstrain field. Mitchelisch et al. [6,7] and Wang et al. [8] extended the work of Mikata and Nemat-Nasser [3] to ellipsoidal inclusion. Mitchelisch et al. [6] considered constant eigenstrain field, whereas the work of Mitchelisch et al. [7] and Wang et al. [8] pertain to impulse non-uniform eigenstrain field.

The influence of the inter-phase layers between the core inhomogeneity and its surrounding matrix on the static response of the heterogeneous solids has been extensively studied [9–17]. The study of the effects of the interface properties and the dynamic behavior of these materials are important in characterization of the inter-facial microstructures through quantitative non-destructive evaluation methods [18–24]. Along the line of scattered fields by a coated reinforcement the work of Shindo et al. [20,25], Shindo and Niwa [26], and Sato and Shindo [23,27] should be mentioned. The common assumption made by these authors is that the coating and core phases are to be concentric. Their analysis is based on the classical wave-function expansion which has been previously employed by Eringen and Suhubi [28] to determine the elastodynamic fields of a single inhomogeneity.

An alternative analytical treatment is based on the equivalent inclusion method (EIM) which has been widely employed to static problems, but hardly engage with dynamic cases. The well-known Eshelby's EIM [2,29,30] provides the elastostatic fields disturbed by a single ellipsoidal inhomogeneity. Within the framework of the EIM, the embedded inhomogeneity can be replaced by an equivalent inclusion whose elastic properties are the same as those of the matrix, but possesses a proper non-zero distribution of eigenstrain field [31]. It is required that the stress field for both the inhomogeneity and inclusion problems to be identical. This equivalency results in a set of consistency equations necessary for the determination of the unknown homogenizing eigenstrains. The extension of the notion of the EIM to the case of multi-inhomogeneity system is given by Shodja and Sarvestani [12]. Eshelby's theories are focused on the polynomial homogenizing eigenstrains. For the elastostatic problems and polynomial far-field loading, Eshelby [30] has predicted the nature of the disturbance strain field inside the embedded ellipsoidal inhomogeneity. This property is quite striking for certain purposes and not necessarily effective for other cases. Examination of the literature reveals that the concept of the polynomial homogenizing eigenstrains has been widely incorporated by numerous investigators without paying special attention to its effectiveness.

The application of EIM to the dynamic problems has been limited probably due to the associated mathematical complexity. The first attempt was made by Fu and Mura [1] who employed the concept of polynomial eigenstrain and introduced the dynamic EIM (DEIM) to compute the elastic scattered fields of a single spheroidal inhomogeneity subjected to plane time harmonic waves. As an approximate solution, Fu and Mura expanded the scattered displacement field and the unknown eigenstrains in term of polynomial of position vector. By matching the coefficients of like powers in $\mathbf{x}$ a set of simultaneous algebraic equations for the unknown coefficients in eigenstrain polynomial expansion is obtained.

To the best of the authors knowledge, after the work of Fu and Mura there is no further advancements on DEIM. The present study aims to extend the DEIM to the rather complex topological situation where the coating and the fiber are eccentric. This problem is solved for a time harmonic incident SH-wave. This work departs from the conventional analysis and considers a special form of the homogenizing eigenstrain field, suitable for the wave problem under consideration. Its efficacy is evident in the analysis and the results which follow. Utilizing the dynamic Green's function associated with the wave equation for an isotropic elastic medium, an analytically challenging integration over the multi-inhomogeneity system is encountered.

The proposed theory is used to revisit the special cases addressed in the literature rigorously. Moreover, the influence of variableness of the coating thickness on the scattered field is studied.

2. Statement of the problem

Consider a cylindrical inhomogeneity Ω with circular cross section of radius R_0 embedded in an infinite medium. The inhomogeneity is separated from the surrounding matrix D by a variable thickness inter-phase layer Ψ whose interface with the matrix has radius $R_0 + h$. The origin of the Cartesian coordinate and the corresponding polar coordinate system are set at the center of the fiber Ω. To keep the symmetry with respect to x_1-axis, it is assumed that the eccentricity between the centers of Ω and $\Omega \cup \Psi$ denoted by Δ is along x_1-direction. All the regions are assumed to be isotropic and elastic with perfect bonding interface conditions. The shear modulus and mass density of the matrix are shown by μ and ρ, respectively. The corresponding quantities for the fiber and coating are μ_1, ρ_1 and μ_2, ρ_2, respectively.

The three phases: fiber; coating; and matrix described above form a double-inhomogeneity system. It is subjected to an incident anti-plane time harmonic shear wave (SH), $u_3^i(\mathbf{x}, t)$ which is polarized along x_3 and is assumed to propagate in the x_1-direction. At any field point, the displacement $u_3(\mathbf{x}, t)$ can be expressed as

$$u_3(\mathbf{x}, t) = u_3^i(\mathbf{x}, t) + u_3^s(\mathbf{x}, t), \tag{1}$$

where u_3^s represents the scattered field due to the presence of the double-inhomogeneity. u_3^s has to be evaluated in such a way that $u_3(\mathbf{x}, t)$ satisfies the wave equation