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Detecting changes in coupling with Granger causality method from time series with fast transient processes

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HIGHLIGHTS

- To study non-stationary time series the Granger causality must be adapted to data.
- The instant of the evolution operator change can be found using the adapted method.
- One can detect if the oscillation properties changed due to the coupling or not.

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ABSTRACT

The ability of the Granger causality method to detect directed coupling between subsystems of a complex system in a moving time window is investigated on etalon oscillators. In particular, the time series consisting of alternate stationary regimes characterised by the different amplitude and shape of oscillations with fast transient processes between these regimes are considered, with similar transitions being possible due to changes either in the coupling or in the individual properties of subsystems. Two popular approaches to surrogate times series generation are used to check the significance of the method results. Two model structures: the standard linear and the special non-linear adapted to data are implemented.

The Granger causality method using the model structure adapted to data is shown to be significantly advantageous in detecting coupling directionality and the instant time of the regime change than the standard linear method, while in some cases the sensitivity and the specificity of the adapted approach are insufficient.

1. Introduction

Investigation of transition from one typical regime to another in complex systems, composed from a number of subsystems, is a fundamental task, because such systems are a corner-stone of modern scientific conceptions. For instance, brain is considered to be composed of large parts: cortex, cerebellum, thalamus, hippocampus, etc., which consist of smaller areas such as different thalamic nuclei or cortical layers. Another example is the Earth climate, which is considered to be composed of individual, but related phenomena such as El-Niño and the North-Atlantic oscillation.

The evolution of such complex systems is commonly observed through measurement of time series from its subsystems. Through measuring the signals of individual subsystems one can try to find out whether the changes in a complex system are result of the changes in individual properties of its subsystems, or they occur due to the changes in the coupling between different subsystems. To answer this question one can use the existing methods based on construction of empirical forecasting models, which are adapted to work in a moving time window. These methods are actively applied in the neurophysiology [1–5] and climatology [6,7]. Among the most popular are the different kinds of Granger causality technique [8–11], information based measures [12,13], a partial directed coherence [14], and approaches based on modelling phase dynamics [15,16]. The main idea of these approaches is similar, and in some cases they can be shown to be completely equal [17].

The time varying Granger causality [18] seems to be very promising for the investigation of non-stationary time series of complex systems due to its significant advantages. First, it allows to determine the coupling directionality, which is not possible with simple measures such as correlation function, coherency function and mutual information. Second, Granger causality demands the

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series of less length (both in number of data points and in number of oscillations) than the transfer entropy or the phase dynamics methods. This fact makes it possible to analyse the coupling in a comparatively short moving time window, so the dependency of the coupling on time can be estimated.

The approaches proposed in [19–21] are advantageous in comparison with the pairwise techniques since they can distinguish between the direct and mediate coupling if all the necessary signals are provided. However, these methods demand a lot of data and are not very suitable for the short and non-stationary series because a large number of coefficients has to be estimated. To decrease the model dimension, and consequently the number of coefficients in [22] it was proposed to use only the primary variables rather than all measured ones (transfer entropy estimates were considered). The principle problem of multivariate techniques is that all variables, considered to make a significant impact on the network dynamics, have to be measured. If some of them are hidden, which often occurs in real experiments (e.g. in the neurophysiology a lot of brain areas are anatomically connected), distinguishing between a directed and a mediate coupling becomes unreliable.

The main shortcoming of the Granger causality approach in application to the task of diagnostics of time-related changes in complex system is that it is heavily based on model construction, therefore time series are assumed to be stationary, at least for a length of a moving window. In some recent applications of the methods similar to Granger causality to the neurophysiology [23,24] the improved dual Kalman filter is combined with the renormalised partial directed coherence (that can be treated as a Granger causality resolved in the frequency domain) and linear phase space modelling to assess the coupling varying in time. In general, such an approach is declared to be applicable to non-linear, non-stationary noisy data of arbitrary nature. However, the practical evidence of that is not completely clear, since originally [24] its efficiency was demonstrated on 4 coupled autoregressive processes of first order with additive Gaussian noise.

The transfer entropy and the partial directed coherence became so popular in comparison with the straightforward Granger causality approach (as implemented in [9,10,25]) due to the heavy dependence of this straightforward approach on the structure of the model and on a choice of the type and the number of nonlinear functions. However, this problem can be at least partially solved using statistical criteria to choose polynomial order (or number and type of other basis functions) and model dimension (e.g. Schwarz criterion [26] (BIC) or Akaike criterion [27]), and taking into account signal properties, while constructing state vector, as it was demonstrated in [28]. So, in order to apply Granger causality to experimental data, it is important, first, to understand, how the non-stationarity of an investigated series can affect the method efficiency. This task is divided into two: to measure method sensitivity to changes in parameters of individual subsystems, and to measure its sensitivity to changes in coupling between them. The first step to this task solution can be done by investigating the series with fast transient processes, separating relatively long stationary stages. The investigation of applying the Granger causality to such series is the goal of the current paper.

In the frames of the formulated goal, the following questions were addressed in the numerical experiment using specially constructed etalon systems:

1. Does the method always detect coupling directionality? E.g. can it show the coupling to be bidirectional, when is it really unidirectional?
2. In what situation can the method sensitivity be insufficient, e.g. due to the inappropriate account of non-linearity, as it was previously mentioned in [9,29]?
3. Usually surrogate time series are constructed to estimate the significance level of achieved results. How can an approach to surrogates construction affect the method results?
4. How does the method perform with an increase of synchrony level between considered time series? Does it allow to distinguish between situations when the synchrony is a result of an interaction, and when it occurs due to some random factors? Is it possible to understand whether the synchronisation is a result of unidirectional or bidirectional interaction?
5. What is the time resolution of Granger causality coupling estimates? How does the Granger causality method perform, when changes in the individual characteristics, such as mean or variance, delay comparatively to changes in coupling?
6. Can the method distinguish between the same changes in the signal shape and amplitude caused by changes either in the coupling or in individual parameters of subsystems?

Formulated issues are very complex and general, and they cannot be solved within the single work. Therefore we only try to perform an example of an investigation for a certain class of systems and signals in order to reveal the most common method features.

In order to realise the potential advantages of the Granger causality method, one should be very careful with the choice of used models and has to take into account the specifics of experimental data even for stationary series. For instance, an insufficient account of non-linearity leads to a loss in the sensitivity [9,29], too low sampling rate [30], observation noise [31] and an inadequate consideration of the time scales of observed series [32] lead to false positive results, redundant variables lead to underestimation of coupling strength, while synergetic ones—to overestimation [33]. However, sometimes even the most simple models are enough to reveal the coupling, as shown in [3], and the qualitative reproduction of the observed dynamical regime is not necessary to succeed [34], as well as linear models can reveal a coupling between non-linear systems [35]. Therefore two versions of the straightforward Granger causality approach were considered: the standard linear algorithm (as regarded in many papers, e.g. [3,5]) and the adapted method developed in [28] in application to the problem of the coupling estimation between the different brain structures for WAG/Rij epileptic rats.

2. The etalon oscillators and the investigation technique

Oscillators which are well known in non-linear dynamics were decided to be used as subsystems. They were modified to demonstrate two different regimes: irregular oscillations with a low amplitude (*regime* 1) and more regular oscillations with a higher amplitude (*regime* 2). Coupling was implemented in a special manner to provide a possibility to switch between these regimes either by changing individual parameters of subsystems or by changing coupling intensity, while both of these ways lead to the same changes in shape and amplitude of oscillations. Ensembles of up to four subsystems were considered.

The etalon oscillator of the first type was a stochastic oscillator with a threshold excitation (a variation of the van der Pol oscillator) with a Toda potential (1):

$$\frac{dx_i}{dt} - \left(r_i - x_i^4 + k_i(t)x_j^2\right)\frac{dx_i}{dt} + 1 - e^{-x_i} = \xi_i(t), \quad (1)$$

where i is a current oscillator number, j is a driving oscillator number, $k_i(t)$ is a variant in time coupling coefficient, r_i is a coefficient of non-linearity, and $\xi_i(t)$ is a realisation of normal white noise. Coefficients $r_i \in [-0.14; -0.07]\ \forall i$, which correspond to a stable fixed point attractor in absence of noise.