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A mathematical modeling of pulsatile flow in carotid artery bifurcation

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ABSTRACT

Real carotid arteries are elastic and perfused with a non-Newtonian fluid. The blood flow is pulsatile characterized by the reverse flow regions at the non-divider wall of internal carotid artery and external carotid artery. A coupling effect between fluid flow and elastic deformation is introduced. In addition we have calculated volumetric flow rate, pressure gradient and Impedance due to pulsatile flow in elastic carotid artery. We analyzed the flow at various locations in common carotid artery and internal carotid artery for different frequencies 60, 90, 120 pulse/min and Reynolds number 500 using a compliance mismatch model of carotid artery. Reverse flow of blood increases the impedance and should not be discarded. The problem has been analyzed mathematically and the agreement between present results and the available literature was found to be quite satisfactory.

1. Introduction

Detailed knowledge of the flow patterns in arteries and especially in bifurcation is of high clinical interest. A bifurcation of clinical interest is the carotid artery bifurcation where the common carotid divides into the external and the internal carotid arteries. The latter artery generally shows a widening in its proximal part called carotid sinus. Atherosclerotic lesions are often located proximally in the sinus, on the non-divider wall. The regions of high shear are found at the divider wall of the bifurcation while regions of low shear and recirculation occur at the non-divider wall, especially in carotid sinus [9]. Thus the detailed knowledge of flow behavior is necessary.

Typically, in elastic models, a considerable reduction in shear stress is found [16]. Computer simulation of blood flow and vessel mechanics has been done in human realistic carotid artery bifurcation [1,11,18,24]. The results are obtained by taking vessel wall elasticity and positive peripheral resistance into account, Doppler flow wave forms can be reproduced numerically [20]. Qualitative flow investigations on bifurcation under pulsatile flow condition are thus of increasing importance. During the relaxation period (the diastolic phase), the flow has strong local convective elements that play a role in blood flow behavior include the elasticity or distensibility of the vessel wall.

In contrast with the distensibility of the wall, pulsatile flow of non-Newtonian fluid has been studied in distensible models of human arteries [6,7,13,17,21,23]. The study of pulsatile flow and atherosclerosis had been done by various authors [4,8,10]. Laser-Doppler anemometer and MRI techniques are being used to investigate the complex blood flow pattern during pulsatile flow in healthy and stenosed carotid artery bifurcation [5,12,22].

To our knowledge pulsatile flow in terms of impedance using deformation of walls of carotid artery bifurcation has not yet been studied. The impedance is a function of position but not a function of time. In our own study, we solved analytically

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Navier–Stokes equations and deformation equations with appropriate boundary conditions. A quantitative analysis for the important quantities is performed at the end of the paper through graphical display of the results and their discussion illustrates the applicability of the present model.

2. Mathematical model

2.1. Elastic model

Blood consists of a suspension of cells in an aqueous solution called plasma which is composed about 90% water and 7% protein. Blood is neither homogeneous nor Newtonian but the flow of blood in large blood vessels is usually modeled as a Newtonian fluid [14,19]. Thus, blood is considered to be Newtonian fluid (no great differences in velocity profiles for the non-Newtonian case were observed [25]) with density $\rho = 1050\ \mathrm{kg/m^3}$ and kinematic viscosity $\upsilon = 4 \times 10^{-6}\ \mathrm{m^2/s}$. Plasma in isolation may be considered Newtonian with viscosity of about 1.2 times that of water. For whole blood, the effective viscosity was found to be shear rate dependent. The Power-Law equation, $\tau = \mu e^n$, found to hold good for strain rates between 5 and 200 $\mathrm{s^{-1}}$ [14].

Fig. 1 illustrates the model of the carotid artery bifurcation and Fig. 2 illustrates the wall movement or deformation of the model used in this study. The vessel axial velocity profile v_z is pulsating and the radial velocity v_r is zero at inlet. For convenience, the model is described using cylindrical polar coordinates r, θ, z with axis of z lying along the central axis of vessel. In an elastic model, the high pressure end distends more than low pressure end and the diameter of vessel becomes deformed because of the variation in pressure [6]. Hence due to the deformation of the wall, velocity vector and pressure gradient has a radial component. Due to the problem symmetry, the flow is independent of θ. Hence assuming,

$$v_r = v_r(r,z,t), \quad v_\theta = 0, \quad v_z = v_z(r,z,t), \quad p = p(r,z,t). \tag{1}$$

Then, unsteady two-dimensional Navier–Stokes equations will describe the motion of the fluid [14] used to study the flow in the model as

$$\rho\frac{\partial v_r}{\partial t} = -\frac{\partial p}{\partial r} + \mu\left(\frac{\partial^2 v_r}{\partial r^2} + \frac{1}{r}\frac{\partial v_r}{\partial r} - \frac{v_r}{r^2} + \frac{\partial^2 v_r}{\partial z^2}\right) \tag{2}$$

$$\rho\frac{\partial v_z}{\partial t} = -\frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r}\frac{\partial v_z}{\partial r} + \frac{\partial^2 v_z}{\partial z^2}\right) \tag{3}$$

and equation of continuity

$$\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial z} + \frac{v_r}{r} = 0. \tag{4}$$

If u_r, u_z are components of the deformation of material of the wall and τ_{rr}, $\tau_{r\theta}$, τ_{rz}, $\tau_{\theta\theta}$, $\tau_{\theta z}$, τ_{zz} are components of symmetric stress tensor, then, the dynamical equations of elasticity becomes

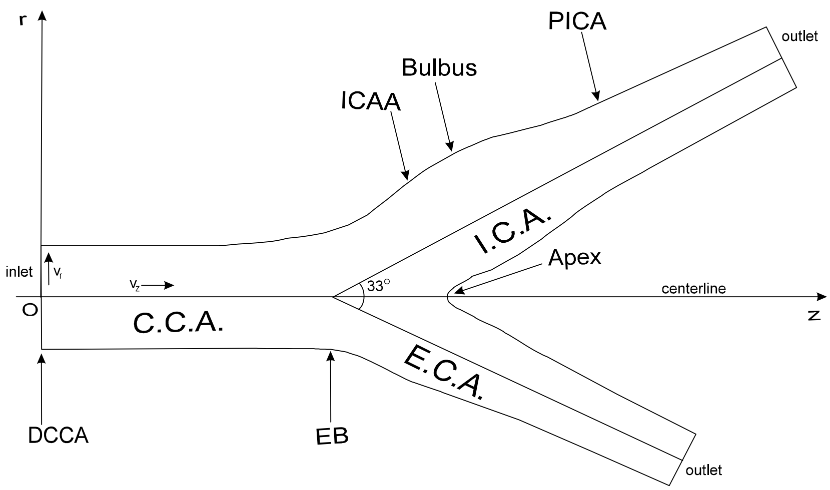

Fig. 1. The model of the carotid artery bifurcation showing different positions.