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Dynamics and stability of a discrete breather in a harmonically excited chain with vibro-impact on-site potential

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ABSTRACT

We investigate the existence and stability of discrete breathers in a chain of masses connected by linear springs and subjected to vibro-impact on-site potentials. The latter are comprised of harmonic springs and rigid constraints limiting the possible motion of the masses. Local dissipation is introduced through a non-unit restitution coefficient characterizing the impacts. The system is excited by uniform time-periodic forcing. The present work is aimed to study the existence and stability of similar breathers in the space of parameters, if additional harmonic potentials are introduced. Existence–stability patterns of the breathers in the parameter space and possible bifurcation scenarios are investigated analytically and numerically. In particular, it is shown that the addition of a harmonic on-site potential can substantially extend the stability domain, at least close to the anti-continuum limit. This result can be treated as an increase in the robustness of the breather from the perspective of possible practical applications.

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1. Introduction

Discrete breathers (DBs) have long been a subject of both theoretical analysis and experimental studies [1–3]. In [4], DBs of the sine–Gordon type are analyzed for various coupling strength values, including the no-coupling limit, showing noteworthy features in terms of existence, stability, bifurcation types, mobility and interaction, and exhibiting properties similar to some of those exhibited by Hamiltonian systems. In [5], magnetic meta-material breathers are analyzed with special emphasis on the weak coupling limit and with stability and mobility investigated for both energy-conserving and dissipative systems. In [6], a model with quartic nonlinearity is analyzed from the perspective of spontaneous creation and annihilation of DBs due to thermal fluctuations, exhibiting the features of stochastic resonance, such as, for example, non-monotonic dependence on noise. A seminal experimental work [7] investigated stability exchange between different localized modes in forced-damped coupled pendula and their possible relation to dislocation dynamics.

In the majority of theoretical studies related to DBs, the considered models are Hamiltonian. Still, in many applications the damping cannot be neglected, and in order to maintain the DB, one should compensate it by some kind of direct or parametric external forcing [3]. Many of the DBs observed in experiments exist in damped systems and should be maintained by some external forcing.

Lack of Hamiltonian structure radically changes the properties of the DBs. To name just one point, instead of a continuous family of localized solutions, one expects to obtain a discrete set of attractors. Accordingly, many of the methods devised for computation and analysis of Hamiltonian DBs are not applicable in forced-damped systems. Recently, it was demonstrated that one can derive exact solutions for DBs in vibro-impact chain models. Such lattices have been investigated analytically both for the Hamiltonian case [8] and for the forced-damped case [9].

In both cases, representation of the nonlinearity, responsible for the localization effect, with the help of the impact conditions, turned out to be advantageous, both for the derivation of an analytic solution and for the stability analysis. To simplify the numeric simulations, in [10], a method is suggested for modeling impact conditions by smooth potentials for both symmetric and single-impacts scenarios. By application of group theory techniques, one can derive smoothened potential and dissipation terms, which rigorously mimic the non-elastic impacts in a limit of large smoothening exponent. The obvious advantage of smoothened impact conditions is the ability to incorporate them into an explicit, stable integration scheme, such as the backward Euler scheme, for example. An inevitable (although, perhaps, acceptable) shortcoming of the method is in that it makes the equations stiff in finite intervals, in finite proximity of the impact constraints. Another shortcoming is the relative complexity of a linear stability analysis of a solution obtained for the smooth problem, relatively to

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the case where genuine impact conditions are imposed. A beneficial approach could be the use of a combination of impact and smooth potentials—performing a linear stability analysis with respect to the impact-potential representation, as argued to be advantageous in [11,12], integrating with the impact scheme when it converges and with the smoothened scheme where there seems to be an instability and it is to be determined whether it arises from the physics or from the integration algorithm.

The present work is in a sense a continuation of [9], where exact expressions for the displacements of the masses in an infinite chain were obtained in the form of a convergent Fourier series. Moreover, for every value of the (dimensionless) link stiffness, a range of amplitudes of the time-harmonic excitation was found, for which a localized breather exists. Interestingly, it was found that no solution corresponding to a phonon-emitting breather could exist. Linear stability analysis based on Floquet theory was performed, utilizing the method of [13]. Three noteworthy features were revealed. First, it was found that for a large-enough value of the dimensionless link stiffness (smaller than the maximum value corresponding to breather existence), one observes loss of stability by delocalization. Second, for low enough link stiffness, there exists a critical value of the excitation amplitude smaller than the critical value corresponding to the limit of existence of the breather, at and above which loss of stability by symmetry breaking takes place. Third, it was found that the delocalization instability sub-domain boundary is non-monotonous with respect to the link stiffness (or the excitation amplitude).

The motivation for the present investigation is two-fold. First, we would like to explore an additional feature of the system that may be reflecting a state of affairs more commonly encountered in practice. For instance, the harmonic part of a uniform on-site potential may represent the effect of weak, non-dissipative coupling to the environment. Second, the extension of the parameter space could supply more information about generic bifurcations and stability of the DBs.

The structure of the present paper is as follows. In Section 2, the model system is discussed and exact expressions for DBs are derived. In Section 3, detailed characteristics of the solution are derived for the case of single-harmonic excitation. In Section 4, the problem of the existence of localized breathers is explored and existence charts in the parameter-space are presented and discussed. In Section 5, linear stability analysis is performed. In Section 6, the equations of motion of the system are integrated numerically for periodic boundary conditions, in order to validate the analytic solution, to check the effect of the boundary conditions and to verify the stability picture. Section 7 is devoted to concluding remarks.

2. Description of the model and analytic treatment

We consider an infinite system of N identical masses, connected by linear elastic springs, each having dimensionless rigidity γ, subjected to harmonic on-site potentials with dimensionless rigidity κ and excited by a time-periodic spatially uniform external loading force $F(t)$, having a period of 2π. The equation of motion for the displacements $u_n(t)$ in this case takes the following form:

$$\begin{aligned}&\ddot{u}_n + (2\gamma+\kappa)u_n - \gamma u_{n+1} - \gamma u_{n-1} = F(t),\\ &|u_n| \le 1, \quad \forall n \in \mathbb{Z}.\end{aligned} \tag{1}$$

We suggest that each oscillator is subjected to rigid symmetric vibro-impact non-elastic constraints at distances ± 1 from the equilibrium positions of the oscillators. Each impact results in an abrupt change of the velocity of the impacting particle. The formal general expression for this can be written as follows:

$$\dot{u}_n|t = \phi^+ + \pi n = U\left(\dot{u}_n|_{t=\phi^- + \pi N}, u_n|_{t=\phi^- + \pi N}\right), \quad \forall\, n, N \in \mathbb{Z} \tag{2}$$

where ϕ represents the time phase lag between the external forcing and the impacts, and the impact function U is to be specified later. At this point we limit ourselves by seeking only those solutions that correspond to strongly localized breathers, when only one particle experiences impact. Hence, we assume that the impact conditions are fulfilled only for the zeroth mass, namely $|u_n| \le 1$ is replaced by $|u_n| < 1\ \forall\ |n| \in \mathbb{N},\ |u_0| \le 1$. We then eliminate the nonsmooth bounding condition by representing it as an external loading force, following [9]:

$$\begin{aligned}&\ddot{u}_n + (2\gamma+\kappa)u_n - \gamma u_{n+1} - \gamma u_{n-1}\\ &\quad = F(t) + 2p\delta_{n0}\sum_{j=-\infty}^{j=\infty}\delta(t-\phi+\pi(2j+1))\\ &\qquad - \delta(t-\phi+2\pi j)\end{aligned} \tag{3}$$

where $2p$ stands for the change in the linear momentum of the zeroth mass due to a single impact incident. $\delta(x)$ is the Dirac-delta function.

As the external forcing is spatially uniform, the solution may be decomposed into a uniform and a non-uniform part:

$$u_n = v_n + f(t) \tag{4}$$

where the uniform part satisfies the equation:

$$\ddot{f}(t) + \kappa f(t) = F(t) \tag{5}$$

the general solution of which is:

$$f(t) = \kappa^{-1/2}\int^t \sin[\kappa^{1/2}(t-\tau)]F(\tau)\, d\tau. \tag{6}$$

Substitution of (4) and (6) into (3) gives an equation for $v_n(t)$:

$$\begin{aligned}&\ddot{v}_n + (2\gamma+\kappa)v_n - \gamma v_{n+1} - \gamma v_{n-1}\\ &\quad = 2p\delta_{n0}\sum_{j=-\infty}^{j=\infty}\delta(t-\phi+\pi(2j+1)) - \delta(t-\phi+2\pi j).\end{aligned} \tag{7}$$

Expanding the right-hand side of (7) into a cosine Fourier series yields:

$$\begin{aligned}&\ddot{v}_n + (2\gamma+\kappa)v_n - \gamma v_{n+1} - \gamma v_{n-1}\\ &\quad = -4p\pi^{-1}\delta_{n0}\sum_{l=0}^{\infty}\cos[(2l+1)(t-\phi)].\end{aligned} \tag{8}$$

The equation of motion in (8) leads to the following dispersion relation for $v_n(t)$:

$$\omega(\zeta) = \sqrt{\kappa + 2\gamma(1-\cos\zeta)} \tag{9}$$

where ζ is a wavenumber. Hence, a solution may in general be phonon-emitting and contain harmonics corresponding to propagating frequencies in the strip: $\sqrt{\kappa} \le 2l+1 \le \sqrt{\kappa+4\gamma}$.

Consequently, we decompose Eq. (8) into two localized and one propagating part, producing the following equations:

$$\begin{aligned}&\ddot{\hat{v}}_n + (2\gamma+\kappa)\hat{v}_n - \gamma \hat{v}_{n+1} - \gamma \hat{v}_{n-1}\\ &\quad = -\frac{4p}{\pi}\delta_{n0}\sum_{l=0}^{\lfloor\sqrt{\kappa}/2-1/2\rfloor}\cos[(2l+1)(t-\phi)]\end{aligned} \tag{10}$$

$$\begin{aligned}&\ddot{\tilde{v}}_n + (2\gamma+\kappa)\tilde{v}_n - \gamma \tilde{v}_{n+1} - \gamma \tilde{v}_{n-1}\\ &\quad = -\frac{4p}{\pi}\delta_{n0}\sum_{l=\lceil\sqrt{\kappa}/2-1/2\rceil}^{\lfloor\sqrt{\kappa+4\gamma}/2-1/2\rfloor}\cos[(2l+1)(t-\phi)]\end{aligned} \tag{11}$$

$$\begin{aligned}&\ddot{\bar{v}}_n + (2\gamma+\kappa)\bar{v}_n - \gamma \bar{v}_{n+1} - \gamma \bar{v}_{n-1}\\ &\quad = -\frac{4p}{\pi}\delta_{n0}\sum_{l=\lceil\sqrt{\kappa+4\gamma}/2-1/2\rceil}^{\infty}\cos[(2l+1)(t-\phi)]\end{aligned} \tag{12}$$

$$v_n = \hat{v}_n + \tilde{v}_n + \bar{v}_n. \tag{13}$$

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