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Thermo-galvanometric instabilities in magnetized plasma disks

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ABSTRACT

In this work, we present a linear stability analysis of fully-ionized rotating plasma disks with a temperature gradient and a sub-thermal background magnetic field (oriented towards the axial direction). We describe how the plasma reacts when galvanometric and thermo-magnetic phenomena, such as Hall and Nernst–Ettingshausen effects, are taken into account, and meridian perturbations of the plasma are considered. It is shown how, in the ideal case, this leads to a significant overlap of the Magneto-rotational Instability and the Thermo-magnetic one. Considering dissipative effects, an overall damping of the unstable modes, although not sufficient to fully suppress the instability, appears especially in the thermo-magnetic related branch of the curve.

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1. Introduction

The dynamical problem of matter falling on the surface of a central astrophysical object dates back to the work of Kuiper, in 1941 [1]. Long before the actual observation of the phenomenon, in that work was inferred that matter would have slowly spiraled inward, thus forming a rotating disk around the star. From there, accretion disk physics and dynamics have been thoroughly studied, mainly due to their connection to crucial phenomena such as Gamma Ray Bursts [2] and Active Galactic Nuclei [3], as well as due to their peculiar observed luminosities, several orders of magnitude higher than those from the corresponding stars. In order to appreciate the relevance of this emission, we emphasize how a compact X-binary disk can emit up to $\mathcal{O}(10^{37})$ erg s^{-1}, whereas the Sun shows a four-orders of magnitude lower luminosity. The problem is that such an efficient energy extraction and transport to the outer layers of the disks has no simple physical interpretation. The reason lies in the fact that the outflow of energy cannot be explained by using the kinetic viscosity of the disk, but instead this phenomenon requires the plasma to be turbulent in order to trigger an effective viscosity.

It has been shown that a linear instability can be triggered in a pure hydrodynamical disk in differential rotation, but only if the vorticity of the plasma (angular momentum per unit of mass) decreases outwards [4]. However, this is a requirement generally not fulfilled by astrophysical systems. The idea that a background magnetic field could be an effective source of enhanced transport is one already presented by Lynden Bell [5,6] and Shakura and Sunyaev [7,8], but it was with the works by Balbus and Hawley [9–12] that the magnetic field could be directly linked to turbulence via the linear instability that it causes in the plasma flows. This is called Magneto-rotational Instability (MRI) and it holds for a differentially rotating thin disk as far as an arbitrarily small magnetic field is involved in the problem. Such an instability is nonetheless affected by specific restriction. First of all, MRI is suppressed for perturbation having sufficiently small wave-length. Moreover, assuming a vertical magnetic field, the disk is actually stable for perturbations propagating in the radial direction only. Finally, for the instability to take place, the following condition must be verified: $v_A^2 > 6v_s^2/\pi^2$, in which $v_A = B/\sqrt{4\pi\rho}$ is the Alfvén velocity of the plasma and $v_s = \sqrt{5P/3\rho}$ the (adiabatic) sound speed (here, B, P and ρ stand for the disk magnetic field, gas pressure and density, respectively): this means that the magnetic field has to be sub-thermal, *i.e.*, in terms of the β plasma parameter we get $\beta > 1$ (we remind that for an adiabatic equation of state, it results $\beta \equiv 8\pi P/B^2 = 6v_s^2/5v_A^2$).

Since the stability emerges in the small β and spacial scale region, we investigate if other collisional contribution to the generalized Ohm can induce new kind of instabilities in such regimes, by also characterizing their absolute or convective nature [13,14]. A similar approach, in the case of axially propagating perturbations, has already been carried out in [15], analyzing an analogous yet substantially different physical configuration. In fact, in our study,

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we consider also a non-zero thermal transport in the plasma, thus the energy equation has to be included in the dynamical system and we succeed in fixing a Thermo-magnetic Instability (TMI), generalizing the analysis in [16]. Moreover, our calculation stands for all spatial scales, holds for radial perturbations too, and also for arbitrarily large values of the magnetic field, *i.e.*, it does not depend on β (note, however, that the choice of the plasma parameter actually affects the magnitude of the instability).

In particular, we analyze the case of local meridian perturbations of the disk parameters by taking into account the galvanometric and thermo-magnetic contributions to the system, which corresponds to include the Hall and Nernst terms in the generalized Ohm law, respectively. In defining how exactly these modifications affect the dynamics of the instabilities, we start by considering, in a non-dissipative environment, the two effects separately and then both of them along with the thermo-electromotive force the plasma is subject to. While the first case results to be not much different from the pure MRI, the second shows an overlap of the MRI absolute modes with the thermo-magnetic ones, thus extending the instability to all scales. It is worth noting how the convective instability results still dominant for large spatial scales too. In fact, the complex and conjugated solutions outline a significant increase of the growth rate as the wave number decreases, finding its cutoff in the natural size of the disk as well as in the validity region of the considered local approximation. We complete our analysis by considering all the above mentioned collisional effects, in the presence of dissipative terms (namely, the electrical and the thermal conductivities). We show how, in such a Thermo-galvanometric Instability (TGMI) scenario, the perturbed system is subjected to a general damping, thus stabilizing a considerable range of the allowed wave-vector values. We shall outline how MRI and TMI are somewhat complementary.

The paper is organized as follows. In Section 2, we present the set of the basic equations which describe the local behavior of the perturbed system. In Section 3, after a brief description of the approximations used and of the main features of MRI and TMI, respectively, we focus on the different kinds of absolute instabilities generated from the presence of each thermo-magnetic effect, in the ideal case of meridian perturbations. A discussion on the emerging convective (propagative) instabilities is presented in the corresponding subsection. In Section 4, the analysis on the role of the dissipative terms in the configuration is developed in the limit of large, as well as, small β parameters. Concluding remarks follow.

2. Basic formalism

The fundamental equations of the accretion disk structure are basically those of MHD, but considering that Hall and Nernst effects modify the expressions of the electric field $\mathbf{E}$ and the heat density flux vector $\mathbf{q}$. This results in changing the frozen-in and energy conservation laws; hence, the heat density flux vector and the generalized Ohm law become

$$\mathbf{q} = \alpha_{\parallel} T\mathbf{j}_{\parallel} + \aleph T\mathbf{B}\times\mathbf{j} - \chi_{\parallel}(\nabla T)_{\parallel} - \chi_{\perp}(\nabla T)_{\perp}, \tag{1}$$

$$\mathbf{E} + \frac{1}{c}\mathbf{v}\times\mathbf{B} = \frac{\mathbf{j}_{\parallel}}{\sigma_{\parallel}} + \frac{\mathbf{j}_{\perp}}{\sigma_{\perp}} + \Re\mathbf{B}\times\mathbf{j} + \alpha_{\parallel}(\nabla T)_{\parallel} + \aleph\mathbf{B}\times\nabla T, \tag{2}$$

respectively, where we have adopted the following notation: $\mathbf{B}$ is the magnetic field (bold stands for vector while $B = |\mathbf{B}|$), χ is the thermal conductivity, σ is the electrical conductivity, $\mathbf{v}$ is the plasma speed, $\mathbf{j}$ is the plasma current density, T is the plasma temperature; the subscripts $\parallel$ and $\perp$ indicate components parallel and perpendicular to the magnetic field lines (in cylindrical coordinates $[r, \phi, z]$). $\Re$, $\aleph$ and α stand for the Hall, Nernst and thermo-electromotive coefficients, respectively, and their approximate expression can be derived using kinetic theory [17] to obtain

$$\Re = -\frac{1}{ecn}, \qquad \aleph = -\frac{\nu_{ei}}{\sqrt{2\pi}\; mc\omega_{Be}^2}, \qquad \alpha_{\parallel} = \frac{1}{e}\left(\frac{\mu}{T} - 4\right), \tag{3}$$

in which e, m are the (positive) electron charge and mass, c is the speed of light, n and k_B are the number density and the Boltzmann constant, respectively; the electron–ion collisional frequency ν_{ei} and the electron gyrofrequency ω_{Be} are defined, denoting with L_e the Coulomb logarithm, by:

$$\nu_{ei} = \frac{4\pi e^4 L_e n}{m^{1/2}\, T^{3/2}}, \qquad \omega_{Be} = \frac{eB}{mc} \tag{4}$$

while the chemical potential μ is described by the following relation

$$\mu = T\ln\left[nh^3\,(2\pi mT)^{-3/2}\right]. \tag{5}$$

It is important to stress that we have intentionally neglected the orthogonal term of the thermo-electromotive force in the Ohm law, *i.e.*, $\alpha_{\perp}(\nabla T)_{\perp}$, since in [16] it has been shown that such a term is negligible with respect to the Nernst contribution. In particular, the expression of $\alpha_{\perp}$ is [17,18] $\alpha_{\perp} \simeq 0.36(\nu_{ie}/\omega_{Be})^2$, while the Nernst coefficient can be rewritten from Eq. (3) as $\aleph \simeq 10^{10}(\nu_{ie}/\omega_{Be})^2$ for typical parameter values of the accretion disk (here, we have set $T = 10^4$ K and $n = 10^9$ cm^{-3}). We recall that the adopted expressions for the kinetic coefficients are only valid in the limit of sufficiently strong magnetic fields, *i.e.*, $\omega_{Be} \gg \nu_{ee}$ (here, ν_{ee} denotes the electron–electron collisional frequency).

Eqs. (2) and (1) have to be combined with Maxwell equations (Faraday and Ampere laws) and the conservation law of internal energy for a one-atom perfect gas [19,20]:

$$\nabla\times\mathbf{E} = -\partial_t\mathbf{B}/c, \tag{6}$$

$$\nabla\times\mathbf{B} = 4\pi\mathbf{j}/c, \tag{7}$$

$$\partial_t P = -\tfrac{5}{3}P\,(\nabla\cdot\mathbf{v}) - \tfrac{2}{3}\nabla\cdot\mathbf{q}, \tag{8}$$

respectively, note that displacement currents are taken here to be negligible since the fluid motion is non-relativistic.

The analysis is implemented at a fixed distance $r = \bar{r}$ from the central body of mass M and the accretion disk is assumed to be thin, *i.e.*, the half-depth $H(r)$ verifies the inequality $H(\bar{r}) \ll \bar{r}$. The local equilibrium configuration is described by a first-order perturbation of the MHD equations described above and we denote the background with $(\ldots)_0$ and the fluctuations with $(\ldots)_1$. A generic variable A is thus perturbed near $\bar{r}$ as $A = A_0(\bar{r}) + A_1(r)$, with $A_1 \ll A_0$. It is important to stress that the effects associated with $\mathbf{q}$ are neglected in the background dynamics being determined by gravity only. Moreover, we are dealing with small scale perturbations and the radial variation of the zeroth-order quantities can be effectively frozen to a given fiducial radius.

The zeroth-order configuration is determined for an adiabatic equation of state $P_0 \sim \rho_0^{5/3}$ (where $\rho_0(\bar{r}, z)$ is the disk mass density). The background radial momentum conservation (which reduces to the balance of the gravitational force with the centripetal one) locally fixes the Keplerian nature of the disk angular frequency as $\omega_k = \sqrt{GM/\bar{r}^3}$. On the other hand, the zeroth-order vertical local equilibrium determines the gravostatic profile of decay for the mass density (and for the pressure) as the vertical coordinate increases. The equilibrium between the pressure and the vertical gravitational force simply results into the background profile [4] $\rho_0(\bar{r}, z) = \tilde{\rho}[1 - z^2/H(\bar{r})^2]^{3/2}$, where $H^2 = 2\tilde{P}/\tilde{\rho}\omega_k^2$ (pressure being $\tilde{P} \sim \tilde{\rho}^{5/3}$, with $\tilde{\rho} = $ const.). In this work, the z-dependence is however disregarded in each background variables in view of the thinness of the disk ($|z| \leq H(\bar{r}) \ll \bar{r}$), thus reducing the mass density to $\rho_0 = \tilde{\rho}$, the pressure to $P_0 = \tilde{P}$ and the temperature to

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