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Nonlinear day-to-day traffic dynamics with driver experience delay: Modeling, stability and bifurcation analysis

Xiaomei Zhao [a,b], Gábor Orosz [c,*]

[a] School of Traffic and Transportation, Beijing Jiaotong University, Beijing, 100044, China

[b] MOE Key Laboratory for Urban Transportation Complex Systems Theory and Technology, Beijing Jiaotong University, Beijing, 100044, China

[c] Department of Mechanical Engineering, University of Michigan, Ann Arbor, MI 48109, USA

HIGHLIGHTS

- Time delays and nonlinearities are introduced into day-to-day traffic models.
- The linear and nonlinear stability of the equilibrium is analyzed.
- Normal forms are derived for the Flip and Neimark–Sacker bifurcations.
- Domains of bistability are determined by numerical continuation.
- Delays extend the bistable region and increase the period of oscillations.

GRAPHICAL ABSTRACT

Day-to-day updating rules for cost and flow with driver experience delay τ:

$$\mathbf{c}_{t+1} = \alpha\, C(\mathbf{f}_{t-\tau}) + (1-\alpha)\,\mathbf{c}_t\,,$$
$$\mathbf{f}_{t+1} = \beta\, F(\mathbf{c}_{t+1}) + (1-\beta)\,\mathbf{f}_t\,.$$

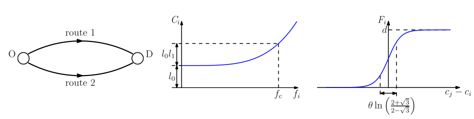

Figure 1: **Left**: a two-route traffic network. **Middle**: nonlinear cost function. **Right**: network loading function.

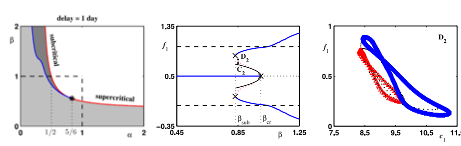

Figure 2: **Left**: global stability chart with linearly stable (light gray), bistable (dark gray), and unstable (white) domains. **Middle**: bifurcation diagram for $\alpha = 1/2$ with subcritical Neimark-Sacker bifurcation. **Right**: stable and unstable tori in state space.

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ABSTRACT

In day-to-day traffic assignment problems travelers' past experiences have important impact on their cost prediction which influences their route choice and consequently the arising flow patterns in the network. Many travelers execute the same trip in every few days, not daily, which leads to time delays in the system. In this paper, we propose a nonlinear, discrete-time model with driver experience delay. The linear stability of the stochastic user equilibrium is analyzed by studying the eigenvalues of the Jacobian matrix of the system while the nonlinear oscillations arising at the bifurcations are investigated by normal form calculations, numerical continuation and simulation. The methods are demonstrated on a two-route example. By applying rigorous analysis we show that the linearly unstable parameter domain as well as the period of arising oscillations increase with the delay. Moreover, delays and nonlinearities result in an extended domain of bistability where the stochastic user equilibrium coexists with stable and unstable oscillations. This study explains the influence of initial conditions on the dynamics of transportation networks and may provide guidance for network design and management.

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* Corresponding author. Tel.: +1 734 763 2769.

E-mail addresses: xmzhao@bjtu.edu.cn (X. Zhao), orosz@umich.edu (G. Orosz).

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1. Introduction

Traffic assignment models have been widely used for transportation planning and network design. These models assign demands for origin–destination (OD) pairs and generate the flow pattern on the whole road network. A fundamental concept in assignment problems is the user equilibrium proposed by Wardrop [1], which is the Nash equilibrium for transportation networks [2]. At this state, all used routes on the same OD pair have the same travel cost, while the unused ones have equal or higher cost. The concept of user equilibrium has been extended to deal with a more realistic driver behavior by incorporating uncertainty, leading to the so-called stochastic user equilibrium [3]. This can be viewed as an optimal state for transportation networks and many papers discussed its existence and uniqueness. However, traffic systems that are based on dynamic route adjustment do not necessarily converge to the equilibrium, even when it is proven to exist. Therefore, stability and bifurcation analysis, which addresses whether and how the flow evolves in time, is essential for understanding the traffic behavior.

Day-to-day traffic assignment models are suitable for analyzing the time evolution of the traffic flow on networks, due to their flexibility of accommodating a wide range of behavior rules, levels of aggregation, and traffic models. These models describe how the flow and the related cost (e.g., travel time) change on the network from one day to another. Day-to-day dynamics have attracted significant attention since the seminal work of Horowitz [4], where a two-route scenario was considered and the stability of the equilibrium was explored. In general, day-to-day dynamics can be formulated as deterministic processes or as stochastic processes [5]. In the deterministic framework, it is assumed that travel cost can be perceived perfectly [6–8]. In particular, Smith [6] assumed that the route switch rate is proportional to the difference of travel cost on the routes, while Friesz et al. [7] proposed a day-to-day model that captures both the dynamics of route flows and OD demands. Zhang and Nagurney [8] used a minimum norm projection operator to model the adjustment of day-to-day route flows. All of these papers considered continuous time. However, Watling and Hazelton [9] pointed out that the continuous-time approach had two limitations: (i) continuous-time trip adjustment is not plausible in reality, and (ii) homogeneous population assumptions require additional dispersion modules.

In discrete-time day-to-day models, travelers' route choice is updated daily, in accordance with daily changes in traffic flow, and it is assumed that each trip is executed at most once a day. Specifically, Nagurney and Zhang [10] discretized their continuous model and applied Euler's method to solve the obtained discrete-time dynamical system. He and Liu [11] proposed a "prediction–correction" framework for modeling discrete-time deterministic day-to-day traffic evolution and also gave sufficient conditions for global stability of the equilibrium. They also calibrated their model for a real network with an unexpected disruption. Bie and Lo [12] investigated the stability of user equilibrium pursued by a day-to-day adjustment process and provided tools to determine the stability of the equilibrium and to estimate its domain of attraction. Stochasticity has also been introduced into day-to-day assignment processes, including models formulated as Markov processes [13,14], and models assuming that route choice probabilities depend on the experienced travel times [4,15–17]. In the latter case, the obtained nonlinear dynamical systems are in fact deterministic, since the route choice probabilities are considered to be deterministic functions of averaged quantities.

Travelers forecast their travel costs on different routes by mixing their own experience with information from other sources (e.g., their friends' experience, broadcast, traffic information center etc.), and they make routing decisions according to their inertia and forecasted cost. A common assumption in day-to-day studies is that travelers make the same trip daily. However, in realistic traffic scenarios, many commuters go to one place on some days and to another place on others, so they execute a particular trip in every few days (instead of every day). Recent research provides evidence for the non-daily trips of travelers. By analyzing data collected about multiple days of travel, one can observe variability in travel behavior because people's needs and desires vary significantly in time [18]. By analyzing 149 individuals, Hanson and Huff [19–22] and Raux et al. [23] pointed out that a one-day pattern is not representative of a person's route of travel. Stopher and Zhang [24] classified all trips into twelve tour types, and concluded that a particular tour happens typically two or three times a week, which means that drivers make decision based information that is 2–3 days old. In addition, some traffic policies may bring such delays into the decision making for the entire driving population: in urban areas they often apply restrictions based on parity of the license plate number which forces drivers to make their trips on every other day.

These findings suggest that travelers' experiences are a couple of days old when they choose a route which introduces time delays into the system. Such *driver experience delays* play a key role in determining the arising traffic patterns. As day-to-day models are constructed at the population level, all parameters in this setup, including the delay, represent population averages, that is, heterogeneities in driver behavior are omitted. However, even after taking the population average the system parameters may still vary in time stochastically. The deterministic models considered in this paper describe the mean dynamics of these processes with time-independent parameters. Our goal is to understand how self-excited oscillations of different periods may arise in these systems without external periodic forcing (i.e., without incorporating demand fluctuations). To explore such behavior we vary the system parameters, including the delay, and detect when qualitative changes occur in the dynamics. Such ideas have been coined by Horowitz [4] and Cascetta and Cantarella [25], but no formal analysis has been carried out to understand the effects of the delay on the linear and nonlinear dynamics.

In this paper, for the first time, we utilize state-of-the-art analytical and numerical techniques from dynamical systems theory to investigate the linear and nonlinear stability of the equilibrium when varying delays and other system parameters. This extends the results by Cantarella and Veloná [26] and Cantarella [17] obtained for the linear behavior of non-delayed models. Our generalized model reveals physical phenomena that have not yet been explained before. We show that, apart from extending the linearly unstable domain, the delay also increases the period of arising oscillations significantly so that it can become much larger than the delay itself. Moreover, applying normal form calculations and numerical continuation, we determine regions of bistability in parameter space where the equilibrium coexists with stable and unstable oscillations. In order to quantify the effects of initial conditions we estimate the domain of attraction of the equilibrium by calculating the amplitude of the unstable oscillations. Finally, we remark that larger the delay is the more complex the nonlinear dynamics may become as the number of coexisting oscillations also increases with the delay.

Prior work on vehicular platoons show that time delays can significantly influence the dynamics of transportation systems and change the arising large-scale patterns [27–30]. In platooning problems, delays occur due to driver reaction time that is typically in order of seconds and can significantly affect the nonlinear oscillations with periods in the order of minutes. In day-to-day traffic models, driver experience delays are in the order of 1–3 days and the corresponding nonlinear oscillations have periods of the order of 5–7 days. Nevertheless, the qualitative dynamics are

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