



Analysis, simulation and visualization of 1D tapping via reduced dynamical models



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HIGHLIGHTS

- A vertically tapped particle column has dynamics approximated by that of a ball.
- The dynamics can be further approximated by iterating a planar diffeomorphism.
- Simulations show the approximations are good predictors of the column dynamics.
- The efficacy of the approximations is also supported by advanced visualizations.

ARTICLE INFO

Article history:

Received 30 September 2013

Received in revised form

22 December 2013

Accepted 30 January 2014

Available online 7 February 2014

Communicated by V.M. Perez-Garcia

Keywords:

Newtonian models

Center of mass model

Discrete dynamical model

Simulations

Visualizations

ABSTRACT

A low-dimensional center-of-mass dynamical model is devised as a simplified means of approximately predicting some important aspects of the motion of a vertical column comprised of a large number of particles subjected to gravity and periodic vertical tapping. This model is investigated first as a continuous dynamical system using analytical, simulation and visualization techniques. Then, by employing an approach analogous to that used to approximate the dynamics of a bouncing ball on an oscillating flat plate, it is modeled as a discrete dynamical system and analyzed to determine bifurcations and transitions to chaotic motion along with other properties. The predictions of the analysis are then compared – primarily qualitatively – with visualization and simulation results of the reduced continuous model, and ultimately with simulations of the complete system dynamics.

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1. Introduction

Continuing our work in [1], we delve even more deeply into the dynamics of a vertically tapped column of particles; this time exploring how certain “averaged” approximations can yield useful information about the dynamics of the complete system such as the identification of a key dimensionless bifurcation parameter related to the acceleration of the tap that signals period-doubling cascades and transitions to chaos. The underlying idea or theme in using such approximate models is that the loss in accuracy may be

more than compensated for by enhanced amenability to detailed mathematical analysis. In particular, we focus on the dynamics of the center of mass of the particle configuration as it evolves under the action of gravity and the periodic tapping force applied via the floor (bottom) of the configuration, which can be approximated by a two-degree-of-freedom system that is essentially ‘equivalent’ to a single ball driven by the oscillatory motion of the floor (as borne out by qualitative comparisons with simulations of the complete system). Naturally, one cannot expect that the motion of the center of mass is capable of illuminating many of the more subtle aspects of the dynamics of the complete system; however, it is plausible that transitions to chaotic regimes can be rather well predicted by the reduced system. The reason for this is that chaotic motions of individual particles would typically not cancel one another in the averaging that defines the position of the center of mass, and the dynamics of the center of mass cannot possibly be chaotic

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<http://dx.doi.org/10.1016/j.physd.2014.01.009>

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unless the same is true for at least one of the particles in the complete system. What we found to be most surprising is that the reduced (center of mass) dynamics appears to be a reasonably good bellwether for other types of changes in the qualitative dynamics such as period-doubling bifurcations and is rather useful for characterizing other dynamical phenomena for the complete system such as the existence of periodic orbits of arbitrarily large period as well as (what appear to be) strange attractors.

We begin in Section 2 with a continuous one-dimensional dynamical model for the vertical column of particles (assuming a modified Walton–Braun type interaction law with different spring constants for loading and unloading [2,3] and an impulse-like oscillatory motion for the floor). The governing equations for the motion of the column of particles are then expressed as a system of second-order ordinary differential equations (ODEs) in the usual Newtonian way (and the initial equilibrium configuration of the particle stack is derived) and this is recast as a system of first-order equations in the standard manner. In these systems, the role of inelasticity (which causes dissipation in the dynamics) is highlighted and quantified by a parameter that is directly related to the coefficient of restitution, which is the more familiar measure of energy loss in the particle–particle and particle–floor interactions. It is interesting and instructive to compare our modeling and analysis with such related investigations as [4–19].

In Section 3, we use the equations of motion of the complete system to determine the reduced governing equations for the dynamics of the center of mass. Taking full advantage of Newton's third law of motion, we show that these reduced equations make it possible to uniquely determine the dynamics of the mass center by a single second-order ODE if the motion of the particle nearest the floor is known. Whence, the assumption of a reasonable relationship between the motion of the particle nearest the floor and the center of mass (which becomes more accurate as the number of particles increases) enables us to approximate the motion of the center of mass by a second-order ODE that is equivalent to a single ball bouncing on the oscillating floor—albeit a ball with mass equal to the total mass of particles in the full system that is acted upon by a suitably modified gravitational force. It should be noted here that the apparent relationship between the bouncing ball and oscillating granular column dynamics was observed about a decade ago, empirically justified, investigated in some detail and more or less confirmed via simulation by Luding et al. [20,21] and Brennen et al. [22]. An inescapable and quite interesting inference that one perforce draws from this equivalence is that the dynamics of the center of mass is at least as rich and varied as that of a ball bouncing on a vibrating plate (as delineated in such studies as [23–26]).

Next, in Section 4, we further simplify our reduced continuous model for the evolution of the center of mass using two discrete dynamical system approximations that rather accurately portray certain key features of the motion of the reduced center of mass model. The first discrete dynamical systems model is developed along the lines formulated by Holmes [24], and also turns out to be basically equivalent to the *standard map* [24,27–29]. We perform an in-depth analysis of dynamics of this model that reveals that there is an acceleration-(or energy-) like non-dimensional parameter that governs transitions from regular motions to period-doubling bifurcations and ultimately to chaotic dynamics; it is the same parameter that has been shown to play a pivotal role in bifurcations of bouncing-ball dynamics and (via simulation and experiments) the motion of the complete stack of particles.

Section 5 is devoted to wide-ranging simulations of the dynamics of a tapped column of a large number of particles (computing the trajectories of the individual particles) and the corresponding motion of the center of mass of the configurations. These simulations – performed using a very effective granular

dynamics based code that has been refined and upgraded over several years (cf. [1,7,15,30,31]) – are focused on observing how the complete system and center of mass reductions respond to changes in several important parameters, both physical and non-dimensional, with an eye toward comparisons with predictions from the analytical investigation in the preceding sections. It is found that there is rather good agreement between the analytical and computational results.

In addition to establishing agreement between the analytical and simulation aspects of our investigation, in Section 6 we include comparisons of the reduced center of mass motion results with conclusions obtained using dynamical visualization techniques such as those described in [32–35]. In particular, appropriate phase space visualization methods are used on the continuous second-order ODE model selected to approximate the dynamics of the center of mass of the configuration. These methods typically involve the identification of several types of coherent phase space structures that have been found to correspond to various dynamical regimes and transitions among them. Comparisons at several levels for numerous parameter values reveal remarkably good qualitative agreement among the analysis, simulation and visualization approaches, thereby confirming the effectiveness of this tripartite strategy for dynamical investigations.

Finally, in Section 7, we conclude with a succinct summary of the results obtained and their importance in this and related investigations. Moreover, we briefly outline our plans for future studies inspired by our work here—directed mainly at useful alternative reduced models and extensions to higher dimensions and generalization of the systems considered.

2. Newtonian model

First, we derive the equations of motion for a vertically tapped column of particles using Newtonian and Hamiltonian principles assuming that the particle–particle and particle–floor interaction forces are of a modified simplified Walton–Braun (W–B) type (cf. [5,6,12,16,36]). A simplified type of W–B model is employed to avoid the extra calculations associated with the initiation of interactions in progress at any given time. It should be noted that the modification does not alter the particle (mass center) dynamics, because both models yield precisely the same velocities before and after contact (related to the coefficient of restitution).

To be precise, we consider a vertical configuration of N particles p_i , $1 \leq i \leq N$, stacked one above the other starting with p_1 , under the action of gravity – with constant gravitational acceleration g – and interacting inelastically (according to the modified W–B model) with neighboring particles and the rigid bottom. The bottom (floor) of the stack, denoted as y_0 and initially at zero, moves so as to simulate a periodic nearly impulsive force applied vertically to the floor. The floor and particle centers are located, respectively, at the points

$$0 \leq y_0(t) < y_1 < \cdots < y_N \quad (1)$$

in $I := \{y : 0 \leq y\}$, and we assume that the particles have masses and radii $m_1, r_1, \dots, m_N, r_N$, respectively.

We assume that $y_0(t)$ is a periodic function of period $T > 0$ represented as

$$y_0(t) := \begin{cases} a \sin \omega t, & 0 \leq t \leq \pi/\omega \\ 0, & \pi/\omega \leq t \leq T \end{cases} \quad (2)$$

for $0 \leq t \leq T$, where $\pi/\omega \ll T$, and the amplitude a is small and positive. The derivative of y_0 with respect to t , $\dot{y}_0$, is the discontinuous function

$$\dot{y}_0(t) := \begin{cases} a\omega \cos \omega t, & 0 \leq t \leq \pi/\omega \\ 0, & \pi/\omega \leq t \leq T. \end{cases} \quad (3)$$

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