



A new interaction potential for swarming models

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ABSTRACT

We consider a self-propelled particle system which has been used to describe certain types of collective motion of animals, such as fish schools and bird flocks. Interactions between particles are specified by means of a pairwise potential, repulsive at short ranges and attractive at longer ranges. The exponentially decaying Morse potential is a typical choice, and is known to reproduce certain types of collective motion observed in nature, particularly aligned flocks and rotating mills. We introduce a class of interaction potentials, that we call Quasi-Morse, for which flock and rotating mills states are also observed numerically, however in that case the corresponding macroscopic equations allow for explicit solutions in terms of special functions, with coefficients that can be obtained numerically without solving the particle evolution. We compare the obtained solutions with long-time dynamics of the particle systems and find a close agreement for several types of flock and mill solutions.

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1. Introduction

Emerging behaviors in interacting particle systems have received a lot of attention in research in recent years. Topics range from diverse fields of applications such as animal collective behavior, traffic, crowd dynamics and crystallization. Self-organization in the absence of leaders has been reported in several species which coordinate their movement (swarming), and several models have been proposed for their explanation [1–6].

Many of these models are based on zones in which some of 3 basic effects are included: short-range repulsion, long-range attraction, and alignment. These 3-zone basic descriptions have been very popular for modeling fish schools [7–11], starlings [12], or ducks [13,14]. The main modeling issues are if some or all of these effects between agents have to be included and if so, how to incorporate them. Many basic swarming models rely on averaged spatial distance or orientation interactions while recent biological studies point out the importance of nearest-neighbor interactions [15] or anisotropic communication [16]. Mathematicians have started in recent years to attack one of the most striking features of these *simple looking* models: the diversity of swarming states, also called patterns in the biology community, their emergence and stability.

The individual level description of these phenomena leads to certain particle systems, called Individual Based Models (IBMs), with some common aspects. Typically, the attraction–repulsion is modeled through pairwise effective potentials depending on the distance between individuals. An asymptotic speed for particles is imposed either by working in the constrained set of a sphere in velocity space [17–19] or by adding a term of balance between self-propulsion and friction which effectively fixes the speed to a limiting value for large times [20,21]. In this work, we will not include any alignment mechanism. We refer to [22] for a survey on results related to kinetic modeling in swarming.

In Section 2 we will review some of these IBMs, and discuss the appearance of two main swarming patterns: mills and flocks. These patterns are easily observed in particle simulations [21,23] and reported in detail for certain particular potentials, the so-called Morse potentials. We will give a precise definition of flocks and mills as solutions of the kinetic equation associated to the particle systems. Finding the spatial shape of flocks and mills has been numerically reported in the literature, but obtaining analytical results on them has only been done in one dimension for the Morse potential in [24].

In this work, we generalize the strategy in [24] proposing a new interaction potential, that we call Quasi-Morse, to replace the Morse potential. The Quasi-Morse potential coincides with the Morse potential in one dimension and we will show that, it is a suitable extension

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of the Morse potential in $n = 2, 3$. Section 3 introduces Quasi-Morse potentials as fundamental solutions of certain linear PDEs. We will first show that the Quasi-Morse potentials are biologically relevant in essentially the same parameter range as the Morse potentials. Second, we make use of their particular structure to show in our main theorem that flock and mill solutions can be expressed as almost explicit linear combinations of special functions.

Finally, Section 4 is devoted to propose an algorithm to compute the scalar coefficients in the expansion of the flock and mill patterns in terms of the basis functions associated with the Quasi-Morse PDE operators. The strategy uses ideas of constrained optimization methods. We finally compare the results for flocks in 2D and 3D and mills in 2D to particle simulations showing a good agreement. As a conclusion, we demonstrate that the proposed Quasi-Morse potentials are very good alternative to Morse potentials as they share many of their features in the natural parameter range, and at the same time enable explicit computation of the macroscopic density profiles up to numerically determined constants.

2. Swarming: Models and patterns

We will consider a simple second order model for swarming analyzed in [21] consisting of the attraction–repulsion of N interacting self-propelled particles located at $x_i \in \mathbb{R}^n$ with velocities $v_i \in \mathbb{R}^n$ in a host medium with friction, with $n = 1, 2, 3$. Friction is modeled by Rayleigh’s law and as a result, an asymptotic speed for the individuals is fixed by the compensation of friction and self-propulsion. More precisely, the time evolution is governed by the equations of motion

$$\begin{aligned} \frac{dx_i}{dt} &= v_i, \\ \frac{dv_i}{dt} &= \alpha v_i - \beta v_i |v_i|^2 - \nabla_{x_i} \sum_{i \neq j} W(x_i - x_j), \end{aligned} \tag{1}$$

where W is a pairwise interaction potential and α, β are effective values for propulsion and friction forces, see [20,21,25,26] for more discussion. The interaction potential $W : \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}$ is assumed to be radially symmetric: $W(x) = U(|x|)$, $x \in \mathbb{R}^n$. The typical asymptotic speed of the individuals is $\sqrt{\alpha/\beta}$. The Morse potential is defined by taking

$$U(r) = -C_A e^{-r/l_A} + C_R e^{-r/l_R},$$

where C_A, C_R are the attractive and repulsive strengths, and l_A, l_R are their respective length scales. We set $V(r) = -\exp(-r/l_A)$, $C = C_R/C_A$, and $l = l_R/l_A$ to obtain

$$U(r) = C_A \left[V(r) - CV \left(\frac{r}{l} \right) \right].$$

The choice of this potential is motivated in [21] for being one of the simplest choices of integrable potentials with easily computable conditions to distinguish the relevant parameters in biological swarms. In fact, it is straightforward to check that in the range $C > 1$ and $l < 1$ the potential $U(r)$ is short-range repulsive and long-range attractive with a unique minimum defining a typical distance between particles. Moreover, in this regime the sign of the integral of the potential:

$$\mathcal{U} := \int_0^\infty W(x) dx = \mathcal{V}(1 - Cl^n) \quad \text{with } \mathcal{V} := \int_0^\infty V(r)r^{n-1} dr < 0, \tag{2}$$

gives a criterion to distinguish between the so-called H -stable and catastrophic regimes. This condition reads as $Cl^n - 1 < 0$ for the catastrophic case in any dimension n , see [21,27]. This property of the potential is important since it is related to the typical patterns emerging in such systems, as classified in [21].

Flocks, where particles tend to form groups, moving with the same velocity, and milling solutions, where rotatory states are formed are of particular interest and are observed in particle and hydrodynamic simulations [21,28] in $n = 2$. Actually, they typically emerge in the large time behavior of the system of particles (1) in the catastrophic regime $Cl^2 < 1$ with $C > 1$ and $l < 1$. In the same range of parameters, randomly chosen initial data lead also to other patterns such as double mills and flocks [21,29]. However mills are not observed in the H -stable regime $Cl^2 > 1$ with $C > 1$ and $l < 1$ while flocks do.

Assuming the weak coupling scaling [30–33] in which the range of interaction is kept fixed and the strength of interaction is divided proportionally between particles, we pass to the rescaled formulation:

$$\begin{aligned} \frac{dx_i}{dt} &= v_i, \\ \frac{dv_i}{dt} &= v_i(\alpha - \beta |v_i|^2) - \frac{1}{N} \nabla_{x_i} \sum_{i \neq j} U(|x_i - x_j|). \end{aligned}$$

This system has a well-defined limit as $N \rightarrow \infty$ which can be expressed as a solution of the corresponding mean-field equation:

$$\partial_t f + v \cdot \nabla_x f + F[\rho] \cdot \nabla_v f + \operatorname{div}((\alpha - \beta |v|^2) v f) = 0, \tag{3}$$

with

$$\rho(t, x) := \int f(t, x, v) dv.$$

Here, $f(t, x, v) : \mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}$ is the phase-space density, and $\rho(t, x)$ is the averaged (macroscopic) density. The mean-field interaction is given by $F[\rho] = -\nabla_x W \star \rho$.

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