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Investigation of vertical vibrations of rigid disk on saturated poroelastic stratum under high-frequency excitation

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Abstract

The forced high-frequency vertical vibrations of a rigid circular disk on a saturated poroelastic stratum supported by a rigid rock foundation are studied. An exact formulation is developed for the dual integral equations governing the mixed boundary-value problem of the vertical vibration. By finding an approximate solution to the exact governing dual integral equations, an evaluation of the amplitude response of the rigid disk is presented for the case of a frictionless contact both between the disk and the stratum and between the stratum and the foundation. The influences of parameters regarding the disk and the elastic stratum on the amplitude response of the disk are discussed. It is concluded that the amplitude response of the disk is dependent on the exciting frequency, the depth of the stratum, the Poison's ratio of the solid skeleton, the porosity of the medium and the radius and mass of the disk.

Keywords: Rigid disk; High frequency; Vertical vibration; Amplitude response; Saturated poroelastic stratum

1. Introduction

The soil-structure interaction is a topic of considerable attention in geomechanics [1]. In the classical problems involving dynamic interaction between a loaded structure and an elastic medium, the dynamic force–displacement relationship between a rigid body and a semi-infinite elastic space or an elastic stratum is of great importance. It plays a key role in determining the response of surface structures to dynamic loadings, in particular, underground blasting loading, seismic excitation and machine vibration. Thus, a large number of research work has been devoted to the subject. For instance, Awojobi and Grootenhuis [2], Robertson [3], and Luco and Westman [4] studied vertical vibrations of a rigid body on an elastic half-space. The vertical vibrations of an arbitrarily embedded rigid body were investigated by Pak and Gobert [5]. Tassoulas and

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Kausel [6] and Veletsos and Tang [7] discussed the vibration of a rigid annular body. In these studies, the half-space is regarded as a single-phase elastic solid. However, geomaterials such as soils are often fluid-saturated porous materials, and it may be more appropriate to describe the half-space as a two-phase medium consisting of a solid skeleton with voids filled with water. On account of dissipation of the pore fluid, a coupling exists between the states of stress in the solid and fluid portions of the medium. From the practical viewpoint, this could impact the dynamic response of the system. The dynamics of a square plate and a strip on a saturated elastic medium has been examined by Halpren and Christiano [8] and Kassir and Xu [9]. The vertical, rotatory and horizontal vibrations of a circular surface footing on a saturated elastic half-space were investigated by Kassir et al. [10,11]. Jin [12] studied the vertical vibration of an elastic circular plate on a fluid-saturated porous half-space. Dargush and Chopra [13] and Japon et al. [14] employed a boundary element method for the vibration analysis of a rigid circular foundation and a rigid strip foundation on a poroelastic medium, respectively.

All the works mentioned above only considered the case of a rigid body resting on an elastic half-space or semi-infinite medium. However, it is known that the elastic stratum is a more realistic representation of the actual medium in practice. The practical need to study vibrations of rigid bodies on a stratum rather than the conventional half-space was fully discussed by Awojobi [15] who dealt with the torsional oscillations of a rigid circular body on an infinitely wide elastic stratum. Gladwell [16] considered the same case of torsional oscillations and obtained approximate solutions for low-frequency cases. The mixed boundary-value problems of the vertical vibration of a rigid circular body and of the rocking of a long rigid rectangular body on an infinitely wide elastic stratum was formulated in terms of dual integral equations by Awojobi [17]. Approximate solutions of these equations in low-frequency range were obtained by establishing an equivalent system on a semi-infinite elastic medium. As structures like buildings, dams, and towers are usually of high inertia ratios, the resonances of these structures generally occur in low-frequency range. Therefore, if interest in the problem is limited to the prediction of resonant frequency, the low-frequency solutions would be adequate in most cases. However, in high-frequency range that may be quite apart from a consideration of resonance, the low-frequency solutions often become invalid. To this end, Awojobi [18] considered the high-frequency-factor torsional vibration of a rigid circular body on an elastic stratum and vertical vibration on a half-space.

The aim of the present work is to study the forced high-frequency vertical vibrations of a rigid circular disk on a saturated poroelastic stratum. The mixed boundary-value problem of the vertical vibration is formulated in terms of dual integral equations. The assumption that it is smooth and frictionless both between the disk and the stratum and between the stratum and the foundation is adopted.

2. Basic equations of motion

It is convenient to consider a rigid circular disk of radius a subjected to vertical vibrations at the surface of an elastic stratum consisting of two-phase material in cylindrical coordinates (r, θ, z) located at the center of the disk with the z-axis pointing towards the medium. Neglecting the displacement of the fluid relative to the solid, the displacement equations of motion governing the propagation of waves in the medium can be expressed as [10]

$$V_{\mathrm{c}}^2 \frac{\partial e}{\partial r} + V_{\mathrm{s}}^2 \frac{\partial \Omega}{\partial z} = \ddot{u}_r \tag{1}$$

$$V_{\mathrm{c}}^2 \frac{\partial e}{\partial z} - V_{\mathrm{s}}^2 \left(\frac{1}{r} + \frac{\partial}{\partial r} \right) \Omega = \ddot{u}_z \tag{2}$$

where e designates the dilatation of the medium, u_r and u_z are the displacement components, and Ω stands for the rotation of the medium and

$$e = \frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z}, \quad \Omega = \frac{\partial u_r}{\partial z} - \frac{\partial u_z}{\partial r}$$

In addition, V_{c} and V_{s} denote the velocities of the compressional and shear waves, respectively, which are given by [10]