FINDING DISCRETE BESSEL AND TRICOMI CONVOLUTIONS OF CERTAIN SPECIAL POLYNOMIALS

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In this article, the discrete Bessel and Tricomi convolutions of the Laguerre–Gould–Hopper polynomials are introduced. Some important properties including recurrence relations and operational representations of these families are established. For suitable values of indices and variables, the Laguerre–Gould–Hopper polynomials yield several special polynomials. Consequently, results for the discrete Bessel and Tricomi convolutions of the corresponding special polynomials are also obtained.

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1. Introduction and preliminaries

The hybrid families of special functions are introduced as discrete convolution of the known special functions. The discrete Hermite and Laguerre convolutions $f_n^{(\mathrm{H})}(x,y)$ and $f_n^{(\mathrm{L})}(x,y)$ are defined as [5, p. 57]

$$f_n^{(\mathrm{H})}(x,y) = n! \sum_{k=0}^{[\frac{n}{2}]} \frac{y^k f_{n-2k}(x)}{k!(n-2k)!} \tag{1}$$

and

$$f_n^{(\mathrm{L})}(x,y) = n! \sum_{k=0}^{n} \frac{(-1)^k y^{n-k} f_k(x)}{(k!)^2(n-k)!}, \tag{2}$$

respectively.

Taking the generic polynomials $f_n(x)$ $(n \in \mathbb{N}, x \in \mathbb{R})$ as x^n in definitions (1) and (2), $f_n^{(\mathrm{H})}(x,y)$ and $f_n^{(\mathrm{L})}(x,y)$ reduce to the 2-variable Hermite–Kampé de Fériet polynomials $H_n(x,y)$ [1] and 2-variable Laguerre polynomials $L_n(x,y)$ [9]. In the case, in which $f_n(x)$ is less trivial, one may get new classes of special functions.

The theory of special functions plays an important role in the formalism of mathematical physics. Bessel functions are among the most important special functions with very diverse applications to physics, engineering, and mathematical analysis ranging from abstract number theory and theoretical astronomy to concrete problems of physics and engineering.

The Bessel functions $J_n(x)$ are defined by means of the following generating function [17],

$$\exp\left(\frac{x}{2}\left(t-\frac{1}{t}\right)\right)=\sum_{n=-\infty}^{\infty} J_n(x)t^n, \qquad t\neq 0; \qquad |x|<\infty, \tag{3}$$

and possess the following series form

$$J_n(x)=\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{x}{2}\right)^{n+2k}}{k!\,\Gamma(1+n+k)}, \qquad |x|<\infty, \tag{4}$$

where n is a positive integer or zero and

$$J_n(x)=(-1)^n J_{-n}(x), \tag{5}$$

where n is a negative integer.

The exponential operators play a crucial role in pure and applied mathematics. These operators are the most commonly used operators to treat evolution problems. We recall that [2]

$$\exp(-\alpha D_x^{-1})=\sum_{n=0}^{\infty}\frac{(-\alpha)^n D_x^{-n}}{n!}=\sum_{n=0}^{\infty}\frac{(-\alpha)^n x^n}{(n!)^2}=C_0(\alpha x), \tag{6}$$

where

$$D_x^{-n}\{1\}=\frac{x^n}{n!} \tag{7}$$

and $C_0(x)$ denotes the Tricomi function of order zero. The n^{th}-order Tricomi functions $C_n(x)$ are defined by the series

$$C_n(x)=\sum_{k=0}^{\infty}\frac{(-1)^k\ x^k}{k!\ (n+k)!} \tag{8}$$

and specified by the following generating function,

$$\exp\left(t-\frac{x}{t}\right)=\sum_{n=-\infty}^{\infty} C_n(x)t^n. \tag{9}$$

Special functions of more than one variable have varied applications in physics and engineering. The importance of the generalized Hermite polynomials has been exploited to deal with quantum mechanical and optical beam transport problems [16]. The usefulness of the generalized Laguerre polynomials to treat radiation physics problems such as wave propagation and quantum beam life time in storage rings due to the quantum fluctuations is a well-established fact [18].

Recently, Subuhi Khan and her co-authors presented a systematic study of certain hybrid classes of special polynomials, see for example [14, 15]. We recall that the