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International Journal of Engineering Science 46 (2008) 286–292
International Journal of Engineering Science
www.elsevier.com/locate/ijengsci

Short communication

# Effect of inherent anisotropy on acceleration wave speeds in hypoplasticity

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Received 15 March 2007; received in revised form 22 May 2007; accepted 2 November 2007

**Abstract**

Generally, soils possess an inherent fabric with transverse isotropy on the bedding plane. As a consequence, the deformation-stress characteristics show a dependence on the loading direction. Anisotropy is characterized by a vector normal to the bedding plane. In this paper we investigate the speeds of acceleration waves in an inherently anisotropic hypoplastic material. For the incrementally nonlinear hypoplastic constitutive equation the wave speed spectrum is continuous in contrast to discrete spectra in incrementally linear constitutive models. The so-called flutter instability arises when the equation for the wave speeds has complex solutions. In the principal stress space, a surface of first occurrence of flutter instability can be identified by analogy with the failure surface defined by a vanishing stress rate. We analyse the influence of the anisotropy on these surfaces and the change in their shape and symmetries depending on the bedding angle.


*Keywords:* Anisotropy; Hypoplasticity; Wave speed; Acceleration waves; Flutter instability

## 1. Introduction

An anisotropic structure is found in most natural sand deposits due to layerwise sedimentation. Distinction can be made between inherent and induced anisotropy. The former is dictated by the fabric immediately after deposition, while the latter is induced by stressing and straining. In the continuum mechanical approach to describe the inherently anisotropic behaviour of granular materials, constitutive models are usually developed in the framework of elasto-plastic theories [1–7]. Thereby the effect of anisotropy on the elastic and plastic behaviour has to be treated separately, often leading to intricate formulae and a large number of parameters.

In this paper we study a hypoplastic constitutive model with anisotropic effects proposed in [8]. A distinguishing feature of hypoplasticity lies in the fact that the deformation is not decomposed into an elastic and a plastic part. Neither a yield surface nor a flow rule are needed to formulate the constitutive equation, but both

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0020-7225/$ - see front matter 
doi:10.1016/j.ijengsci.2007.11.001

can be derived from the hypoplastic model. Anisotropic effects are incorporated into the model by the use of an anisotropy operator [9] applied to the nonlinear term of the constitutive equation.

Acceleration waves are an important issue in the theoretical analysis of solids as their existence and properties are closely related to well-posedness, stability, bifurcation and shear band formation. In contrast to the discrete wave speed spectra in elasto-plastic models, the speed spectra for the incrementally nonlinear hypoplastic model are continuous [10,11]. For certain stress states the wave speeds vanish giving rise to stationary discontinuity or shear banding, or become complex, resulting in flutter ill-posedness [12,13]. Inherent anisotropy influences the occurrence of these phenomena and changes the corresponding stress states.

## 2. Anisotropy

A hypoplastic constitutive equation of rate-type can in general be written as [14,15]

$$\dot{\mathbf{T}} = \mathbf{H}(\mathbf{T}, \mathbf{D}), \tag{1}$$

where $\mathbf{T}$ is the stress tensor, $\mathbf{D}$ is the rate of deformation tensor,

$$\mathbf{D} = \frac{1}{2}\left[\frac{\partial \mathbf{v}}{\partial \mathbf{x}} + \left(\frac{\partial \mathbf{v}}{\partial \mathbf{x}}\right)^{\mathrm{T}}\right], \tag{2}$$

and $\mathbf{v}(\mathbf{x})$ is the velocity field. The rate $\dot{\mathbf{T}}$ in (1) stands for the material time derivative of the stress tensor. For simplicity, in the constitutive function we use the material time derivative of the stress tensor instead of the corotational stress rate usually used in hypoplasticity.

A solid is considered isotropic if its mechanical response to loading does not depend on the orientation of the loading in space. Eq. (1) possesses induced anisotropy due to the presence of the stress tensor in the constitutive function. Inherent anisotropy can be introduced in a constitutive equation through additional vectorial or tensorial parameters which reflect the mechanical structure of the solid such as shape and orientation of grains in a granular material or orientation of layers in a layered structure. We consider a material element with transverse anisotropy and refer to a rectangular coordinate system in which the axis $x_1$ coincides with the vector of isotropy $\mathbf{a}$ (Fig. 1). In planes normal to $\mathbf{a}$ the material is assumed to be isotropic. The principal stress axes are defined by referring to another coordinate system in which the axis $x_i'$ is aligned with the directions of the principal stresses $T_i$. We restrict ourselves to the case where the axes $x_3'$ and $x_3$ coincide. The angle between $x_1$ and $x_1'$, called the bedding angle, is denoted by $\theta$. In order to describe inherent anisotropy, constitutive equation (1) has to depend explicitly on the orientation of the bedding plane [8],

$$\dot{\mathbf{T}} = \mathbf{H}(\mathbf{T}, \mathbf{D}, \mathbf{a}). \tag{3}$$

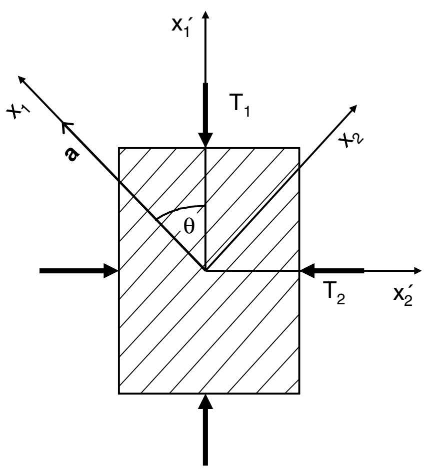


Fig. 1. Coordinate axes and bedding angle $\theta$.