Vol. 80 (2017) *REPORTS ON MATHEMATICAL PHYSICS* No. 3

AN ADVANCED KINETIC THEORY FOR MORPHING CONTINUUM WITH INNER STRUCTURES

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(*Received January 14, 2017*)

Advanced kinetic theory with the Boltzmann–Curtiss equation provides a promising tool for polyatomic gas flows, especially for fluid flows containing inner structures, such as turbulence, polyatomic gas flows and others. Although a Hamiltonian-based distribution function was proposed for diatomic gas flow, a general distribution function for the generalized Boltzmann–Curtiss equations and polyatomic gas flow is still out of reach. With assistance from Boltzmann's entropy principle, a generalized Boltzmann–Curtiss distribution for polyatomic gas flow is introduced. The corresponding governing equations at equilibrium state are derived and compared with Eringen's morphing (micropolar) continuum theory derived under the framework of rational continuum thermomechanics. Although rational continuum thermomechanics has the advantages of mathematical rigor and simplicity, the presented statistical kinetic theory approach provides a clear physical picture for what the governing equations represent.

Keywords: rational continuum mechanics, kinetic theory, morphing continuum.

1. Introduction

The Navier–Stokes equations have been extensively used to study fluid flow physics for several decades. In the fluid mechanics society, it is usually believed that the Navier–Stokes equations are derived from kinetic theory [1] under the assumption that the system of monatomic gases is nearly in a Boltzmann distribution (equilibrium). A system of monatomic gases in equilibrium defines all the physical quantities in the Boltzmann distribution. For example, the Navier–Stokes equations can be derived from the Boltzmann distribution with a first-order approximation applied to the Boltzmann transport equations.

Alternatively, beginning in the early 1960s, Eringen [2, 3], Truesdell [4, 5] and others [6–8] applied rational continuum thermomechanics, with its mathematical rigor, to investigate continuum theories in solids, fluids, mixtures [9, 10] and electrodynamics. The foundation of rational continuum thermomechanics starts with a set of balance laws, including continuity, linear momentum, angular momentum, and energy conservation equations. With the assistance of several axioms, such as objectivity (also known as frame-indifference), memory, etc., and the Coleman–Noll procedure [11] based on the Clausius–Duhem inequality (entropy principle), constitutive equations for a fluid can be derived. The combination of balance laws and constitutive equations for fluids from this approach can also lead to the Navier–Stokes equations [3, 4, 6].

Most of the complex fluid flows contain structures across multiple spatial length scales and are usually dominated by the subscale motions. For example, the rotational eddies characterize the turbulence physics at bulk scale. In addition, most gases are either diatomic or polyatomic. At high altitude or during speed flights, the rotation and vibration of the diatomic/polyatomic molecules causes nonequilibrium flow pheonmena extensively. Those flows are known for not being able to be analyzed by the classical continuum mechanics or Navier–Stokes equations. As a result, the continuum theory used to model these flows with subscale or inner structures shall be categorized as morphing continuum.

Although rational continuum thermomechanics provides mathematical rigor to the theoretical formulation, it leaves the physical meanings of material constants in resulting fluid equations unexplained. It is common for practitioners of the rational continuum thermomechanics approach to interpret the material constants through experiments. In contrast, the kinetic theory approach gives physical meaning to the quantities in the fluid equations by the means of the collision and distribution functions. Due to the lack of mathematical rigor in the kinetic theory approach, Truesdell commented that all equations derived from kinetic theory can be considered only as a class of constitutive equations in rational continuum thermomechanics [12]. Although this comment may be true, the physics hidden in all proposed governing equations derived from kinetic theory can be used to intepretate the same equations using the rigorous procedure of rational continuum thermomechanics. Such is the purpose of this paper.

Despite the successes of the Navier–Stokes equations derived by both kinetic theory and rational continuum thermomechanics, the derivations have been limited to monatomic gases or rather, volumeless point particles. Inspired by the rigor of rational continuum thermomechanics, Eringen was the first to mathematically formulate the micropolar continuum theory with independent inner structures [13, 14]. This work was later expanded by Chen et al. for nonlinear constitutive equations for fluids with electromagnetic interactions [7]. Through the multiscale formulation coupling macroscale and subscale motions, this morphing continuum theory successfully reproduces and explains turbulence physics [15–18], polyatomic gases [19] and microfluids [20, 21] which cannot be explained by the classical Navier–Stokes theories. Nevertheless, a disadvantage of the rational continuum thermomechanics approach is that the physical interpretations of the material constants in MCT are still unknown. Consequently, researchers rely on dimension analysis to decipher the underlying physics. Wonnell and Chen presented a systematic derivation on Eringen's micropolar theory for incompressible turbulence and found a dimensionless parameter differentiating laminar, transition and turbulent flows. One could also rely on advanced kinetic theory to deduce the physical meanings of these MCT material constants. This would be done in a process similar to the one used to derive the Navier–Stokes equations. This newly introduced advanced kinetic theory would go beyond