ON A TWO-PARTICLE BOUND SYSTEM ON THE HALF-LINE

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(*Received August 24, 2016*)

In this paper we provide an extension of the model discussed in [10] describing two singularly interacting particles on the half-line $\mathbb{R}_+$. In this model, the particles are interacting only whenever at least one particle is situated at the origin. Stimulated by [11] we then provide a generalisation of this model in order to include additional interactions between the particles leading to a molecular-like state. We give a precise mathematical formulation of the Hamiltonian of the system and perform spectral analysis. In particular, we are interested in the effect of the singular two-particle interactions onto the molecule.

Keywords: singular interactions, molecule, discrete spectrum, essential spectrum, quantum graph.

1. Introduction

Singular many-particle interactions on general compact quantum graphs were introduced in [3, 4] in order to provide a model for the investigation of many-particle quantum chaos. Based on that, a model of two singularly interacting particles on the half-line $\mathbb{R}_+ = [0, \infty)$ (a simple non-compact quantum graph) was formulated in [10]. More precisely, the Hamiltonian of this model is formally given by

$$H = -\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} + v(x, y)\,[\delta(x) + \delta(y)]\,, \tag{1}$$

$v : \mathbb{R}^2 \to \mathbb{R}$ being some symmetric (real-valued) interaction potential. Due to the properties of the δ-potential we see from (1) that the two particles are interacting only whenever at least one of the particles is situated at the origin. Furthermore, given the support of v is contained in $B_\varepsilon(0)$, i.e. the open ball of radius $\varepsilon > 0$ around $0 \in \mathbb{R}^2$, the two particles are interacting only whenever one particle is situated at the origin and the other is ε-close to it.

From a physical point of view, the interesting property of (1) is that the two-particle interactions are spatially localised onto the origin. Usually, one expects the two-particle interaction to depend on the relative coordinate only. However, as outlined in [10], there are situations where a spatial localisation of many-particle interactions is expected due to certain inhomogeneities of the real physical system to be modelled. In particular, the authors refer to so-called composite wires

from the field of applied superconductivity [7]. Those consist of superconducting parts as well as normal-conducting parts and due to the Cooper pairing effect of superconductivity, the interaction between a pair of electrons in such a wire depends on their corresponding spatial position. In this sense, the Hamiltonian (1) might be used to model a system of two electrons in a wire which is normal-conducting except for a relatively small part at the beginning of the wire which is superconducting. In addition, as described in [8, 9], non-separable quantum two-body problems are quite rarely discussed in the literature although having important applications in condensed matter physics as well as quantum entanglement.

In this paper we are interested in an extension of the model discussed in [10] by adding to (1) an attractive binding potential between the particles leading to a molecular-like state. This was motivated by [11] (see Eq. (5) therein) where the scattering of a two-particle bound system at mirrors is investigated. In particular, the authors are interested in the scattering process given each particle of the molecule is scattered separately at a mirror (each of the mirrors is modelled by a δ-potential). Actually, by allowing for a nonconstant potential v in (1), the model discussed in the following also provides an extension of the model discussed in [11]. In their language, the scattering of one particle of the molecule at its mirror is then no longer independent from the position of the second particle. From a physics point of view, this seems to be a possible scenario.

The paper is organised as follows: In Section 2 we formulate the model and rigorously construct the Hamiltonian of the system via a suitable quadratic form. In Section 3 we perform spectral analysis and describe the essential as well as the discrete part of the spectrum. We prove the existence of an eigenstate below the essential spectrum in the case of vanishing singular interactions and which is due to the geometry of the one-particle configuration space. We then investigate the effect of additional singular two-particle interactions on the spectrum and prove, as a main result, that the discrete part of spectrum becomes trivial given the singular interactions are repulsive and strong enough.

2. The model

In this paper we consider a system of two (distinguishable) particles moving on the half-line $\mathbb{R}_+ = [0, \infty)$ described by the formal Hamiltonian

$$H_b = -\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} + v(x, y)\,[\delta(x) + \delta(y)] \; + V_b(|x - y|), \tag{2}$$

$V_b : \mathbb{R}_+ \to (-\infty, \infty]$ being some binding potential and $v : \mathbb{R}^2 \to \mathbb{R}$ some symmetric interaction potential. For simplicity, we choose V_b to be given by

$$V_b(|x - y|) := \begin{cases} 0 & \text{if} \quad |x - y| \leq d, \\ \infty & \text{else}, \end{cases} \tag{3}$$

$d > 0$ characterising the "size" of the molecule. Due to the presence of the binding potential, the two-particle configuration space has been reduced from $\mathbb{R}^2_+$ to Ω which