CONSISTENT DESCRIPTIONS OF QUANTUM FIELDS

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Study of explicit generalized functions identifies a technical generalization of the Wightman functional analytic axioms that admits realizations of quantum fields with interaction. Scalar field examples are discussed.

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1. Introduction

1.1. Background

Axioms for a topic are motivated by the desire to achieve an overview of conclusions that follow and an assessment of the consistency of the axioms with additional assumptions [1]. Without benefit of nontrivial examples, Arthur Wightman identified functional analytic properties of the vacuum expectation values (VEV) of quantum fields [2, 3] and the consistency of these properties is demonstrated by free fields and related physically trivial constructions. However, despite significant efforts [3–8], demonstrations that quantum fields with nontrivial interaction satisfy the resulting axioms have not resulted.

Constructions of "unconstrained" quantum fields [9] that exhibit nontrivial interaction are reviewed in this note. The unconstrained constructions result in example realizations of quantum fields that satisfy the physical characteristics of relativistic quantum physics and exhibit interaction. These quantum fields are also described by VEV as linear functionals dual to selected function spaces, but the linear functionals are described by a technical generalization of the Wightman functional analytic axioms [9, 10]. The Wightman axioms include principles referred to here as the *physical conditions*: the states of nature are realized as elements of Hilbert spaces and in appropriate instances, the states are physically interpretable as classical particles; scalar products of the states are Poincaré covariant; the fields are locally commutative; and the energy-momenta spectra lie within the closed forward (positive energy) light cone. The admitted example nontrivial realizations satisfy these physical conditions but do not satisfy Wightman's original axiom describing VEV as tempered distributions and, as a consequence, do not necessarily result in Hermitian

field operators. To achieve the goal of example realizations of nontrivial relativistic quantum physics, the Wightman axioms are technically revised and example VEV are derived without reliance on the canonical formalism [11]. Nevertheless, these example realizations exhibit appropriate nontrivial relativistic quantum physics: the states have positive energies; transition amplitudes, associated with the scalar products of states by the Born rule, are Poincaré covariant; the trajectories of isolated, dominant regions in the support of states propagate along trajectories modeled by Newtonian mechanics for $1/r$ potentials [12]; and the plane-wave scattering cross sections are weak coupling asymptotes for the Feynman (Stueckelberg–Feynman–Dyson) series results [9, 10, 13]. This generalized description of the VEV of quantum fields exploits functional analytic methods to enable retention of the established physical characterizations with realizations that exhibit interaction. The development discussed in this review weakens Wightman's original conjecture for the function spaces that are the basis of constructions of rigged Hilbert space realizations.

VEV that satisfy the Wightman axioms and exhibit physically nontrivial interaction remain unrealized despite significant efforts in constructive quantum field theory [3–8]. The lack of demonstrable realizations of relativistic quantum physics is in contrast to the significant phenomenological successes of Lagrangian-based interaction picture methods associated with the Feynman series [11]. Technical concerns with the Feynman series method include the unsuitability of the conjectured unitary equivalence of free and interacting fields demonstrated in Haag's theorem and the appearance of unitarily inequivalent representations of canonical commutation relations (CCR) for unbounded numbers of particles [1, 3, 4].

The discussion in this review emphasizes possibilities for realization of relativistic quantum physics that are not widely considered. A reconsideration of the axioms enables a development of explicit, example realizations for the VEV of fields that model nontrivial relativistic quantum physics and approximate the results from Feynman series. To simplify discussion, the review is limited to a single, neutral, scalar quantum field.

1.2. Another perspective on construction of the free field

In this section, the development of free fields is re-examined with the additional consideration of selecting the space of functions used to label the states and quantum field operators. The technical changes to the Wightman functional analytic axioms [2–4, 14] and the notation of the revision are illustrated in this development of a free quantum field. Hilbert space realizations of relativistic free quantum fields based upon alternative selections for function spaces are contrasted. The convenience of the free field model is that the conjectured Hermitian field operators are realized. The development for a free quantum field is studied for insight into whether Wightman's original selection of the Schwartz functions [15] is necessary. The consideration is whether the Schwartz functions are determined by nature or more convenient alternatives consistent with nature are available.