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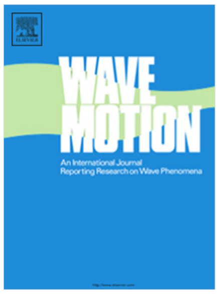

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Abstract

We construct a family of exact planar solitary wave solutions in a two-dimensional lattice. The system under consideration is a scalar two-dimensional extension of a nonintegrable Fermi-Pasta-Ulam problem with a piecewise quadratic potential. The constructed solutions exhibit an anisotropic dependence on the angle of propagation. Through a detailed analysis of explicit solutions, we show that conventional quasi-continuum models fail to fully describe this dependence. However, a truncated series approximation of the constructed solution that includes sufficiently short wavelengths captures this effect quite well.

1 Introduction

Solitary waves have been experimentally observed in many spatially discrete physical systems, from granular materials [5, 6, 35, 40] to electrical networks [27, 29, 39]. These localized coherent waveforms arise as a result of the interplay of nonlinearity and dispersion and transport energy through the system. Since their discovery [52] in the Fermi-Pasta-Ulam (FPU) chain [2, 13, 23], solitary waves in nonlinear Hamiltonian lattices have become the subject of numerous experimental and theoretical studies. In particular, significant progress has been achieved in understanding these waveforms in integrable systems, such as the Toda lattice [42], and in the near-integrable low-energy limit, where their delocalized form is well described by the Korteweg-de Vries (KdV) equation and its analogs in higher dimensions [3, 18–20, 26, 34]. Another well understood limit is that of high-energy waves that exhibit atomic-scale localization [17, 25, 42]. Solitary waves in nonintegrable systems outside the two limiting cases have been primarily investigated using numerical, quasicontinuum and asymptotic methods [7, 10, 15, 16, 21, 22, 47–50].

In a recent work [43], a family of exact solitary wave solutions was constructed for the one-dimensional nonintegrable FPU problem with a piecewise quadratic interaction potential. The series solutions, obtained in [43] in the form of nested approximations corresponding to the series truncations, make the contributions of the progressively smaller wavelengths fully explicit and describe the crossover regime from the KdV-like near-sonic limit to anti-continuum localization. They also constitute a benchmark case that allows one to directly test the validity of various quasicontinuum approximations [48].