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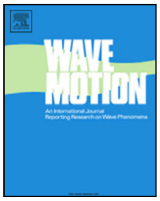

Water wave trapping in a long array of bottomless circular cylinders

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HIGHLIGHTS

- Water wave trapping by long arrays of bottomless circular cylinders is investigated.
- Trapped-modes correspond to the homogeneous solution of the diffraction problem.
- The idea is to examine multi-body arrays as a mechanism for wave energy extraction.
- The solution provided is based on the 'direct' method of approach.
- It is shown that trapped-modes are enhanced by the 'pumping' modes in the moonpools.

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ABSTRACT

This study tries to identify wave trapping situations by engaging and properly combining two well established phenomena: (i) the trapped modes induced by arrays of cylinders and (ii) the pumping trapped modes which are known to occur in moonpools. To this end, the fundamental hydrodynamic boundary value problem for arrays of bottomless cylinders was solved using standard domain decomposition. The method employed expansions of the solutions for the velocity potentials in polar harmonics combined with the eigenfunction expansions technique. The solution sought for the velocity potentials is achieved using the "direct" method of approach which accordingly requires the employment of a sophisticated matrix manipulation process.

The elaboration of the concerned concept was motivated by three basic tasks: (i) to identify whether arrays of truncated and bottomless cylinders indeed preserve the occurrence of Neumann, Dirichlet and near trapped modes, extensively investigated for bottom-seated cylinders; (ii) to examine whether the expected pumping modes in moonpools modify the characteristics of the hydrodynamic resonance regimes (trapped modes) in the open liquid space between the cylinders and vice versa and (iii) to explore the possibility to suggest relevant configurations as parts of integrated mechanisms for practical applications, focusing a fortiori to clusters of hydrodynamically interacting Oscillating Water Columns (OWCs).

The method developed is generic and can be employed for arbitrary configurations of multi-body arrays accommodating bottomless cylinders with uneven geometrical characteristics. Trapped modes are identified numerically as peaks in loading and this fact has been explicitly demonstrated in rows of cylinders. Therefore, the numerical results shown and discussed in the present are based on a specific in-line array that has been investigated in the past for bottom-seated cylinders. The investigated subject, i.e. whether the combined wave trapping induced by the examined configuration could be conceived as an efficient

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water wave power extraction mechanism is approached and discussed through dedicated computations of the free-surface displacements in the moonpools.

1. Introduction

One of the most challenging and intriguing topics in water wave hydrodynamics is the so-called *wave trapping phenomenon*. The necessary condition is to cause continuous scattering of the water waves in a restricted and well-defined region of liquid. Continuous scattering is accomplished between "walls" which apply a Neumann condition, although trapped modes (i.e. eigenvalues leading to wave trapping) can be associated with a Dirichlet condition as well. A typical configuration to investigate wave trapping is the existence of a solid in waveguides, e.g. the vertical walls of a channel [1–13]. In that respect, trapped modes could occur, aside from hydrodynamics, in acoustic waveguides and in the vicinity of electromagnetic gratings. Although different in nature, trapped modes associated with those disciplines are very similar [12].

In three-dimensional problems, such as in hydrodynamical applications, the equivalent of an obstacle, say a vertical cylinder, in a liquid field of finite depth, situated between the walls of a channel, is a row of identical cylinders. In this case, the middle cylinder is the obstacle and the cylinders on either side of the obstacle are the image bodies representing the walls. In relevant situations, trapped modes may occur, that accordingly lead to hydrodynamic resonances and peaks in loading. The trapped modes correspond to specific wavenumbers of the propagating regular waves, which induce the concerned hydrodynamic resonances. Clearly, the phenomenon is correlated with strong wave elevations and wave run-up on the wetted surface of the bodies. That is indeed the outcome of the wave trapping phenomenon that concentrates wave energy in a confined region (in relevant cases the liquid regions between the cylinders) allowing only a small amount of energy to radiate to the far-field. [14–29]. Indeed, the continuous diffraction of the waves in the local vicinities of the modules of the array causes the formation of near standing waves with very large amplitudes. Aside from the theoretical work(s) on the subject, the trapping of waves in multi-column arrangements has been verified through laboratory experiments as well [15].

The practical importance of the concerned phenomenon is substantial, especially for column-based platforms used often in offshore industry such as tension-leg platforms and semi-submersibles operating in deep water fields. The loading induced on these structures is not the only issue of concern. The strong wave elevations and the wave run-up on the columns may reduce, even cancel, the air gap between the underneath area of the platform and the free surface, yielding severe water impacts on the bottom surface of the deck [14,30].

In arrays of cylinders, pure trapping of waves is possible only for infinite arrays. Pure trapped waves are known as Rayleigh–Bloch or edge waves which for particular wavenumbers reduce to trapped mode solutions for a cylinder between two parallel walls having either Neumann or Dirichlet boundary conditions upon them [19]. The Neumann condition is the obvious mathematical compulsion, while the Dirichlet condition corresponds to a fictitious channel where the velocity potential (rather than its normal derivative) vanishes on the walls. Dirichlet trapped modes are known to exist in the acoustic context. Trapped modes beyond the Dirichlet wavenumber are often called near trapped modes which typically result in weak energy radiation. Given that, in reality, there is always an amount of energy (although small) radiated to the far-field, relevant phenomena are often referred as near trapping effects. Admittedly, for a finite row of cylinders all trapped modes can be regarded as near trapped modes (Neumann and Dirichlet included).

Trapped modes were first identified by Ursell [1] in an open channel. Callan et al. [2] used Ursell's method to prove the existence of trapped modes in two dimensional waveguides while Evans et al. [6] showed theoretically the existence of trapped modes for all symmetric cross-sections. The methodology and the elegant formulae reported by Linton and Evans [18] for the water wave diffraction problem by arrays of bottom seated cylinders was the base for several studies on the trapped modes and wave trapping. Evans and Porter [19,20] and Meylan and Eatock Taylor [21], investigated possible trapping and near-trapping effects by means of this method. Maniar and Newman [22] based a great part of their study on the wave trapping by a long array of cylinders (nine in particular), on the concerned "direct" method as well. Evans and Porter [23] extended their efforts to trace possible trapped modes in multiple cylinders in a channel. Grice et al. [24] considered possible near-trapping effects that originate from the interaction of multi-column structures with random waves, while Malenica et al. [31] extended the investigation on near trapping to the second-order in wave steepness.

Most of the studies in wave trapping by arrays of cylinders concern bottom seated cylinders. There is an evident reasoning behind this simplification as bottom-seated and free-surface piercing cylinders allow significant simplification of the diffraction problem given that only the fundamental wavenumber is participating. For any other configuration, such as arrays of truncated, free-surface piercing cylinders the solution of the diffraction problem requires the infinite evanescent modes as well, making a much more complicated system. Arrays of truncated circular cylinders were considered by Wolgamot et al. [27] and Siddorn and Eatock Taylor [28]. The latter study treated a square configuration for both the diffraction and the radiation problems, but the authors did not consider the wave trapping phenomenon. In Wolgamot et al. [27] however, the authors examined wave trapping phenomena for two different arrays, consisting of four and eight cylinders respectively, using the panel method software DIFFRACT.