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The scattering of a scalar beam from isotropic and anisotropic two-dimensional randomly rough Dirichlet or Neumann surfaces: The full angular intensity distributions ☆

Torstein Storflor Hegge [a], Torstein Nesse [a], Alexei A. Maradudin [b], Ingve Simonsen [a,c,d,*]

[a] Department of Physics, NTNU — Norwegian University of Science and Technology, NO-7491 Trondheim, Norway
[b] Department of Physics and Astronomy, University of California, Irvine, CA 92697, USA
[c] Department of Petroleum Engineering, University of Stavanger, NO-4036 Stavanger, Norway
[d] Surface du Verre et Interfaces, UMR 125 CNRS/Saint-Gobain, F-93303 Aubervilliers, France

HIGHLIGHTS

- Wave scattering from randomly rough two-dimensional hardwalls are obtained.
- Rigorous computer simulations are performed.
- The full angular distribution of the scattered intensity is obtained.

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ABSTRACT

By the use of Green's second integral identity we determine the field scattered from a two-dimensional randomly rough isotropic or anisotropic Dirichlet or Neumann surface when it is illuminated by a scalar Gaussian beam. The integral equations for the scattering amplitudes are solved nonperturbatively by a rigorous computer simulation approach. The results of these calculations are used to calculate the full angular distribution of the mean differential reflection coefficient. For isotropic surfaces, the results of the present calculations for in-plane scattering are compared with those of earlier studies of this problem. The reflectivities of Dirichlet and Neumann surfaces are calculated as functions of the polar angle of incidence, and the reflectivities for the two kinds of surfaces of similar roughness parameters are found to be different. For an increasing level of surface anisotropy, we study how the angular intensity distributions of the scattered waves are affected by this level. We find that even small to moderate levels of surface anisotropy can significantly alter the symmetry, shape, and amplitude of the scattered intensity distributions when Gaussian beams are incident on the anisotropic surfaces from different azimuthal angles of incidence.

☆ T.S.H. and T.N. contributed equally to this work.

* Corresponding author at: Department of Physics, NTNU — Norwegian University of Science and Technology, NO-7491 Trondheim, Norway.
E-mail address: Ingve.Simonsen@ntnu.no (I. Simonsen).

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1. Introduction

Scalar wave scattering from non-planar, impenetrable surfaces has been studied extensively in science and engineering [1–3]. Initially, this scattering problem was treated by various approximate methods. Here it suffices to mention geometrical optics or ray-tracing [4,5], and physical optics methods based on the Kirchhoff approximation [6,7].

The earliest nonperturbative calculations of the scattering of a field from a two-dimensional randomly rough surface were the studies of the scattering of a scalar beam, incident from vacuum, on a Dirichlet [8–10] or a Neumann surface [10] carried out by Tran and Maradudin and by Macaskill and Kachoyan. These calculations were based on Green's second integral identity [11]. The integral equations for the source functions, namely the values of the field in the vacuum or its normal derivative, evaluated on the rough surface, were transformed into matrix equations which were then solved by iterative approaches. The amplitudes of the scattered field are expressed in terms of these source functions, and the differential reflection coefficient is expressed through the scattering amplitudes. The differential reflection coefficient (DRC), an experimentally accessible quantity, gives the fraction of the total time-averaged flux incident on the rough surface that is scattered into an element of solid angle about a specified direction of scattering. In scattering from a randomly rough surface it is the average of the DRC over the ensemble of realizations of the surface profile function that is calculated. The result is called the mean differential reflection coefficient (mean DRC). Multiple scattering effects, in particular enhanced backscattering [12], were present in the results for the dependence of the mean DRC for in-plane scattering on the polar angle of scattering and a fixed polar angle of incidence.

Although in the years following this pioneering work several nonperturbative calculations of the scattering of vector fields from impenetrable [13–16] and penetrable [17–22] two-dimensional randomly rough surfaces were carried out, little attention seems to have been directed at rigorous nonperturbative calculations of the scattering of incident beams from Dirichlet and Neumann surfaces perhaps because they are simpler than the scattering problems studied in these references. Nevertheless, the results of these calculations are relevant, for example, in ocean acoustics in the context of the scattering of a sonic wave from a rough ocean floor [23,24].

In this paper we revisit the problem of the scattering of a scalar beam from a two-dimensional randomly rough surface, and investigate properties of the scattered field not considered in the earliest studies of this problem [8–10]. Thus, in addition to presenting results for scattering from surfaces whose profiles are isotropic Gaussian random processes we also present results for the scattering from surfaces whose profiles are anisotropic Gaussian random processes. In addition to the contribution to the mean differential reflection coefficient from the field scattered incoherently in plane, we also present results for the reflectivities of these surfaces and the full angular distribution of the intensity of the scattered field. Moreover, these calculations are carried out by means of improved algorithms that yield accurate solutions of the integral equations arising in the scattering theory without the use of iterative methods of the Sturm–Liouville type or modifications thereof [8–10].

2. Scattering system

The system we consider in this work consists of a medium that supports the propagation of scalar waves without absorption, *e.g.* a liquid, in the region $x_3 > \zeta(\mathbf{x}_\parallel)$, where $\mathbf{x}_\parallel = (x_1, x_2, 0)$ is an arbitrary vector in the plane $x_3 = 0$, and a medium that is impenetrable to scalar waves in the region $x_3 < \zeta(\mathbf{x}_\parallel)$ [Fig. 1]. The surface profile function $\zeta(\mathbf{x}_\parallel)$ is assumed to be a single-valued function of $\mathbf{x}_\parallel$ that is differentiable with respect to x_1 and x_2, and constitutes a stationary, zero-mean, Gaussian random process. It is defined by

$$\left\langle \zeta(\mathbf{x}_\parallel) \right\rangle = 0 \tag{1a}$$

$$\left\langle \zeta(\mathbf{x}_\parallel)\zeta(\mathbf{x}'_\parallel) \right\rangle = \delta^2 W(\mathbf{x}_\parallel - \mathbf{x}'_\parallel), \tag{1b}$$

where the angle brackets here and in all that follows denote an average over the ensemble of realizations of the surface profile function. The quantity δ, the root-mean-square roughness of the surface, is defined by

$$\delta = \left\langle \zeta^2(\mathbf{x}_\parallel) \right\rangle^{\frac{1}{2}} . \tag{2}$$

The function $W(\mathbf{x}_\parallel)$ introduced in Eq. (1b) is the *normalized surface height autocorrelation function*, and has the property that, $W(\mathbf{0}) = 1$. In what follows we will also require the *power spectrum* of the surface roughness, $g(\mathbf{k}_\parallel)$, where $\mathbf{k}_\parallel$ is a two-dimensional wave vector $\mathbf{k}_\parallel = (k_1, k_2, 0)$. The power spectrum is the Fourier transform of the normalized surface height auto-correlation function,

$$g(\mathbf{k}_\parallel) = \int \mathrm{d}^2\mathbf{x}_\parallel \, W(\mathbf{x}_\parallel) \exp\left(-\mathrm{i}\mathbf{k}_\parallel \cdot \mathbf{x}_\parallel\right) . \tag{3}$$

In this work we will assume the following Gaussian form for $W(\mathbf{x}_\parallel)$ [15]

$$W(\mathbf{x}_\parallel) = \exp\left(-\frac{x_1^2}{a_1^2} - \frac{x_2^2}{a_2^2}\right), \tag{4}$$