Wave Motion 80 (2018) 82–90

Contents lists available at ScienceDirect

Wave Motion

journal homepage: www.elsevier.com/locate/wamot

Double degeneration on second-order breather solutions of Maxwell–Bloch equation

Shuwei Xu [a], Jingsong He [b,*], K. Porsezian [c]

[a] College of Mathematics Physics and Information Engineering, Jiaxing University, Jiaxing, Zhejiang 314001, PR China
[b] Department of Mathematics, Ningbo University, Ningbo, Zhejiang 315211, PR China
[c] Department of Physics, Pondicherry University, Puducherry 605014, India

A R T I C L E I N F O

Article history:
Received 10 March 2018
Received in revised form 12 April 2018
Accepted 23 April 2018
Available online 5 May 2018

Keywords:
Self-induced transparency
Maxwell–Bloch equations
Breather solution interactions
Breather solution degeneration
Darboux transformation

A B S T R A C T

The breather solutions of the Maxwell–Bloch equations in a two-level resonant system associated with the self-induced transparency phenomenon are constructed by the Darboux transformation. After constructing the formulas of the second-order breather solutions, the double degeneration and hybrid solutions are studied by the analytical form as well as figures. Our results might be helpful in such application or prevention of the rogue waves from breather solution interactions and degeneration in the nonlinear optical systems associated with the Maxwell–Bloch equations.

1. Introduction

Recently, rogue wave has received lots of attention due to its potential applications in different physical systems of many branches of science, such as photonic crystal fibers [1,2], and ocean [3] and water tanks [3–5], etc. One kind of doubly localized rational solution both in time and space of certain nonlinear equations, such as the Nonlinear Schrödinger (NLS) equation [6–8], the Hirota equation [9,10], the Derivative Nonlinear Schrödinger Equation [11–15], the complex modified Korteweg–de Vries equation [16], the Extended NLS equation [17–20], the two-component NLS equations [21–23], the Davey–Stewartson equation [24] and KP-I equation [25], are presently well accepted as potential prototype nonlinear evolution equations possessing peculiar rogue wave solutions. This rational solution possessing a large amplitude and two hollows reflects the visual character of the rogue wave, i.e., appearing from nowhere and disappearing without a trace [7], and thus is now called the rogue wave solution.

A large number of papers have been published on the relationship between the breather solutions and rogue waves. The pioneering work by Peregrine [6] given that the first-order rogue wave solution of the NLS equation, considered as the Peregrine soliton, is usually expressed in terms of a simple rational algebraic formula and can be constructed from a breather solution through the limit of the infinitely large period. Due to the instability of small amplitude perturbations that are usually chaotic and may contain many frequencies in their spectra, the breather solutions could be generated in different ways. Since the analytic expressions for higher-order solutions are usually cumbersome and admit highly complicated forms, finding the lower-order ones are always important and useful to discuss their intriguing properties. Theoretically, higher-order rogue waves can be generated by the higher-order breathers through a double degeneration [26–30]. The realistic

* Corresponding author.
E-mail addresses: hejingsong@nbu.edu.cn, jshe@ustc.edu.cn (J. He).

https://doi.org/10.1016/j.wavemoti.2018.04.003

problem of the observation of the optical rogue wave in fiber system is to implement the effective collision of the multi-breathers [31]. Based on the NLS equation, one possible way [31] to solve this problem by observing the central profile of the degenerate higher-order breathers instead of the direct observation of the higher-order rogue waves.

Though many exciting results have been reported on rogue waves in few optical systems, it is still interesting and useful to explore the observation of higher-order rogue waves in other integrable soliton equations which is used to model the propagation of the light in different nonlinear optical system by using the above-mentioned way, and thus we need to discuss deeply the double degeneration of the higher-order breathers of the associated modeling equation. The Maxwell–Bloch (MB) equations in a two-level resonant system associated with the self-induced transparency (SIT) is an important completely integrable model in nonlinear optics which explains the pulse propagation through a resonant two-level optical medium [32–34], and thus inspires us to consider its second-order breather solutions and degeneration. McCall and Hahn [35] have proposed a type of optical soliton in a two-level resonant system associated with the SIT phenomenon. Later, Lamb discovered that the phase variation of coherent-optical-pulses in a two-level SIT medium to a reasonable approximation which is frequently described by the MB equations of the following form [36,37]:

$$\begin{aligned} E_\xi &= \rho, \\ \rho_\tau &= -2i\rho\eta + EN, \\ N_\tau &= -\frac{1}{2}(E\rho^* + E^*\rho). \end{aligned} \tag{1}$$

Here, τ and ξ correspond respectively to the time and space coordinates, $E(\tau,\xi)$ is the complex electric field envelope resulting from the interaction between the control laser and the two-level atomic system, $\rho(\tau,\xi)$ is the out-of-phase and in-phase components of the induced polarization, $N(\tau,\xi)$ is the normalized population inversion, the parameter η is the deviation of the transition frequency of the given two-level atom from the mean frequency. Like NLS equation, MB equations are also responsible for many basic developments in soliton theory and widely studied by several authors. In addition to soliton solutions, other types of solutions of MB equations and several generalizations of MB equations are also reported [38], such as Maxwell–Bloch systems with variable coefficients [39–42]. The integrability and soliton aspects of the MB equations are given in Refs. [43–45]. Recently, the SIT phenomenon in a two-level atomic system are profoundly modified by coherent control laser field [46,47]. Moreover, the rogue wave solutions [48,49] of the MB equations are constructed from periodic background solution through degenerate Darboux transformation(DT) [50–57].

The aim of this paper is to study the second-order breather solutions of the MB equations and their double degeneration, which implies the second-order rogue waves. Further, a superposition of two second-order breather solutions may create a hybrid solutions, such as a rogue wave and a breather solution, by means of different settings of the parameters in analytical formulas. This gives a comprehensive understanding of higher-order breather solutions and their hierarchical nature. The structure of this paper is arranged as follows. In Section 2, we provide analytically the determinant representation of the N-order breathers constructed by using the N-fold DT. In Section 3, the second-order breather solutions and their properties such as the interaction and the double degenerations are discussed. Finally, we summarize our main results in Section 4.

2. The *N*-order breather solutions

It is worth noting that the unreduced MB equation of the following form,

$$\begin{aligned} E_\xi &= \rho, \quad E_{1\xi} = \rho_1, \\ \rho_\tau &= -2i\rho\eta + EN, \quad \rho_{1\tau} = 2i\rho_1\eta + E_1 N, \\ N_\tau &= -\frac{1}{2}(E\rho_1 + E_1\rho), \end{aligned} \tag{2}$$

are an integrable system generated by the following compatibility condition

$$U_\xi - V_\tau + [U,V] = 0 \tag{3}$$

of the linear spectral problems [43]

$$\partial_\tau \phi = U\psi = (J\lambda + \frac{1}{2}U_0)\phi, \tag{4}$$

$$\partial_\xi \phi = V\phi = -\frac{1}{4(\eta-\lambda)} iV_{-1}\phi. \tag{5}$$

Here

$$J = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}, \quad U_0 = \begin{pmatrix} 0 & -E \\ E_1 & 0 \end{pmatrix}, \quad V_{-1} = \begin{pmatrix} N & \rho \\ \rho_1 & -N \end{pmatrix}, \quad \phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix},$$

and λ is an iso-spectral eigenvalue parameter. Note that Eq. (2) is equivalent to Eq. (1) with the reduction conditions $E_1 = E^*$, $\rho_1 = \rho^*$.