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On the existence of non-polarized azimuthal-symmetric electromagnetic waves in circular dielectric waveguide filled with nonlinear isotropic homogeneous medium

Yury Smirnov, Eugene Smolkin *

Department of Mathematics and Supercomputing, Penza State University, Krasnaya Str. 40, Penza, 440026, Russia

HIGHLIGHTS

- Phenomenon of non-polarized azimuthal-symmetric electromagnetic wave propagation is considered.
- The problem is formulated with physically realistic conditions.
- An original analytic approach is used to study the problem.
- It is proved the existence of a novel (nonlinear) guided regime.

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ABSTRACT

The propagation of monochromatic nonlinear symmetric hybrid waves in a cylindrical nonlinear dielectric waveguide is considered. The physical problem is reduced to solving a transmission eigenvalue problem for a system of ordinary differential equations. Spectral parameters of the problem are propagation constants of the waveguide. The problem is reduced to the new type of nonlinear eigenvalue problem. The analytical method of solving this problem is presented. New propagation regime is found.

0. Introduction

Circular dielectric waveguide is one of the most simple waveguiding structure which is used in electrodynamics. Theory of propagation of electromagnetic waves in this waveguide is well elaborated in linear case, i.e. when waveguide filled with linear isotropic homogeneous medium [1,2]. It is well-known that there are two types of azimuthal-symmetric electromagnetic polarized waves: TE- and TM-waves [1]. And there is no non-polarized azimuthal-symmetric electromagnetic waves in circular dielectric waveguide filled with linear isotropic homogeneous medium. (We will not consider the case when

* Corresponding author.
E-mail addresses: smirnovyug@mail.ru (Y. Smirnov), e.g.smolkin@hotmail.com (E. Smolkin).

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propagation constants of TE- and TM-waves coincide because this is a very special one and it can be realized only for special parameters of the waveguide).

Consider a circular dielectric waveguide filled with nonlinear isotropic homogeneous medium. We will assume that medium in the waveguide is described by the Kerr law, i.e. permittivity depends on electric field as follows: $\varepsilon = \varepsilon_0\varepsilon_2 + \varepsilon_0\alpha|\mathbf{E}|^2$. In this case two types of azimuthal-symmetric electromagnetic polarized TE- and TM-waves exist also (see [3,4] for TE-waves and [5] for TM-waves). The main goal of the paper is to prove the existence of the new third type of azimuthal-symmetric electromagnetic (non-polarized) waves which do not correspond to the TE- and TM-waves.

From the mathematical point of view, this problem can be considered as two-parameter nonlinear eigenvalue problem [6]. The problem under consideration is reduced to the new type of nonlinear two-parametric eigenvalue problem because there exists only one spectral parameter (compare with [6]). General methods to study such problems have not been developed yet. Below we present rigorous mathematical proof of the existence of such nonlinear hybrid waves. We think that this type of waves can be used in electrodynamics and electronic devices.

1. Statement of the problem

Consider three-dimensional space $\mathbb{R}^3$ equipped with a cylindrical coordinate system $O\rho\varphi z$. The space is filled with isotropic medium with constant permittivity $\varepsilon = \varepsilon_1\varepsilon_0$, where $\varepsilon_0 > 0$ is the permittivity of vacuum. In this medium a circular cylindrical waveguide is placed. The waveguide is filled with isotropic nonmagnetic medium. The waveguide has cross section $\Sigma := \{(\rho,\varphi,z) : 0 \le \rho < r, 0 \le \varphi < 2\pi\}$ and its generating line (the waveguide axis) coincide with the axis Oz. We assume $\mu = \mu_0$, where $\mu_0 > 0$—magnetic permeability of vacuum. The waveguide is unlimitedly continued in the z direction.

Maxwell's equations have the form [7]

$$\begin{aligned} &\operatorname{rot}\tilde{\mathbf{H}} = \partial_t\tilde{\mathbf{D}},\\ &\operatorname{rot}\tilde{\mathbf{E}} = -\partial_t\tilde{\mathbf{B}}, \end{aligned} \tag{1}$$

where $\widetilde{\mathbf{D}} = \varepsilon\widetilde{\mathbf{E}}$, $\widetilde{\mathbf{B}} = \mu_0\widetilde{\mathbf{H}}$ and $\partial_t \equiv \partial/\partial t$. From system (1) we obtain

$$\begin{aligned} &\operatorname{rot}\tilde{\mathbf{E}} = -\partial_t(\mu_0\tilde{\mathbf{H}}),\\ &\operatorname{rot}\tilde{\mathbf{H}} = \partial_t(\varepsilon\tilde{\mathbf{E}}). \end{aligned} \tag{2}$$

We assume that the fields $\tilde{\mathbf{E}}$, $\tilde{\mathbf{H}}$ depend harmonically on time [8]

$$\begin{aligned} &\widetilde{\mathbf{E}}(\rho,\varphi,z,t) = \mathbf{E}^+(\rho,\varphi,z)\cos\omega t + \mathbf{E}^-(\rho,\varphi,z)\sin\omega t,\\ &\widetilde{\mathbf{H}}(\rho,\varphi,z,t) = \mathbf{H}^+(\rho,\varphi,z)\cos\omega t + \mathbf{H}^-(\rho,\varphi,z)\sin\omega t, \end{aligned}$$

where ω is the circular frequency, $\widetilde{\mathbf{E}}$, $\mathbf{E}^+$, $\mathbf{E}^-$, $\widetilde{\mathbf{H}}$, $\mathbf{H}^+$, $\mathbf{H}^-$ are the real functions.

It is clear that

$$\widetilde{\mathbf{E}} = \operatorname{Re}\left\{\mathbf{E}e^{-i\omega t}\right\}, \quad \widetilde{\mathbf{H}} = \operatorname{Re}\left\{\mathbf{H}e^{-i\omega t}\right\},$$

where

$$\mathbf{E} = \mathbf{E}^+ + i\mathbf{E}^-, \quad \mathbf{H} = \mathbf{H}^+ + i\mathbf{H}^-$$

are the complex amplitudes and

$$\begin{aligned} &\mathbf{E} = \left(\mathrm{E}_\rho, \mathrm{E}_\varphi, \mathrm{E}_z\right)^T,\\ &\mathbf{H} = \left(\mathrm{H}_\rho, \mathrm{H}_\varphi, \mathrm{H}_z\right)^T; \end{aligned}$$

where $(\,\cdot\,)^T$ is the transposition operation. Each field component is a function of three space variables.

We suppose that permittivity inside the waveguide Σ has a form

$$\varepsilon = \varepsilon_0\varepsilon_2 + \varepsilon_0\alpha\left(|\mathrm{E}_\rho e^{-i\omega t}|^2 + |\mathrm{E}_\varphi e^{-i\omega t}|^2 + |\mathrm{E}_z e^{-i\omega t}|^2\right), \tag{3}$$

where ε_2 and $\alpha > 0$ are real constants. It is obvious that $|\mathbf{E}e^{-i\omega t}| = |\mathbf{E}|$. This allows us to write down Eq. (2) for the complex amplitudes $\mathbf{E}$, $\mathbf{H}$:

$$\begin{cases} \operatorname{rot}\mathbf{H} = -i\omega\varepsilon\mathbf{E},\\ \operatorname{rot}\mathbf{E} = i\omega\mu_0\mathbf{H}. \end{cases} \tag{4}$$

The complex amplitudes $\mathbf{E}$, $\mathbf{H}$ must satisfy Eq. (4); the continuity condition for the tangential field components on the boundary $\rho = r$; the radiation condition at infinity, where electromagnetic field exponentially decays as $\rho \to \infty$ in the domain $\rho > r$.

The solutions to Eq. (4) are considered for in the whole space.