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Implications in the interpretation of plane-wave expansions in lossy media and the need for a generalized definition

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HIGHLIGHTS

- Plane-wave expansions (PWE) are rigorously introduced only for lossless media .
- For lossy media the PWE and its propagator present unphysical properties.
- We show that a Laplace-based definition solves these issues.
- The PWE definition is no longer unique, and does not allow any physical interpretation.
- This definition is consistent with formula for far-field radiation from PWE.

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ABSTRACT

Plane-wave expansions (PWEs) based on Fourier transform and their physical interpretation are discussed for the case of homogeneous and isotropic lossy media. Albeit being mathematically correct, standard Fourier-based definition leads to nonphysical properties, such as the absence of homogeneous plane waves, lack of dissipation along transversal directions and inaccurate identification of single plane waves. Generalizing the PWE definition using Laplace transform, which amounts to switching to complex spectral variables, is shown to solve these issues, reinstating physical consistency. This approach no longer leads to a unique PWE for a field distribution, as it allows an infinite number of equivalent definitions, implying that the interpretation of the individual components of a PWE as physical plane waves does not appear as justified. The multiplicity of the generalized definitions is illustrated by applying it to the near-field radiation of an elementary electric dipole, for different choices of Laplace cuts, showing the main differences in the generalized PWEs.

1. Introduction

Expanding field distributions onto a Fourier basis is a well-established procedure used for solving problems of radiation and propagation [1–8]. Also known as spectral representation, it provides an interesting and effective framework, as it allows algebraic representations of integro-differential equations, which lend themselves to physical interpretation, since each basis function corresponds to the mathematical description of a plane wave. Field distributions are therefore represented as a linear combination of plane waves, where each one can be propagated through a homogeneous or layered space according to physical laws, whence their being referred to as plane-wave expansions (PWEs).

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It is therefore reasonable that the individual components of a PWE are often considered as physical plane waves, regarding their propagation vectors and amplitudes as physical parameters that accurately describe the way they propagate through a homogeneous medium. While this has a physical foundation in the case of propagative contributions, as recalled in Section 2, the interpretation of PWE has been at the center of controversies. In [9,10], the need to consider portions of a PWE not as necessarily physically consistent at the individual scale was already pointed out for reactive contributions to a PWE, when taken individually. A related issue was reported in [5,11,12], when considering homogeneous contributions in asymptotic expressions.

The problems at interpreting a PWE worsen when lossy media are considered. The introduction of losses is presented in the literature as not requiring any modification to the standard Fourier-based definition. Section 2.2 comments on some inconsistencies that make an intuitive interpretation of a PWE look dubious, e.g., the absence of any homogeneous contribution in case of lossy media. In particular, the PWE can be shown not to provide accurate identification of single plane waves, as opposed to lossless settings.

Section 3 studies an alternative definition of PWE based on Laplace transform. Allowing complex spectral variables, Laplace transform makes it possible again to identify single plane waves from field distributions even in lossy media. The existence of an extended region of convergence for Laplace transform implies that a field distribution does not correspond to a unique PWE, thus leading to an extended family of propagators. All of these propagators yield identical results when propagating field distributions from one plane to another. But each propagator being different it is no longer possible to associate a single common physical meaning to each individual component (or plane wave) across all PWEs.

Numerical examples in Section 5 illustrate the fundamental differences in the generalized PWE depending on the chosen Laplace cut, thus supporting the conclusion that the PWE cannot be interpreted, in lossy media, as composed of physical entities, but should rather be regarded as a mathematical representation. More specifically, there is a case for choosing on purpose alternative definitions when identifying single plane waves that are expected to be homogeneous on physical grounds, such as for asymptotic representations needed for far-field radiation.

2. Plane-wave expansion and field propagation

In the following an $\exp(\mathrm{j}\omega t)$ time dependence will be assumed, and dropped throughout the paper for simplicity, thus working with phasor notation. The background medium is assumed to be homogeneous and isotropic, with a relative permittivity constant ϵ_r and an electric conductivity σ supposed for the time being to be equal to zero, thus with a propagation constant k_o defined as

$$k_o^2 = \omega^2 \mu_o \epsilon_o \epsilon_r. \tag{1}$$

In this section and the next one scalar field distributions are considered for the sake of simplicity, but the ideas discussed directly apply to vector fields as well, as done in Section 5.

2.1. Standard Fourier-based definition

The standard PWE definition is usually introduced by invoking the property of completeness of the Fourier basis [3,4,7,8]. A generic scalar field $u(\boldsymbol{R}, z_o)$ sampled at a plane $z = z_o$ (the scan plane), outside the source region, is projected onto 2D functions of the kind $f(\boldsymbol{R}; \boldsymbol{K}) = \exp(-\mathrm{j}\boldsymbol{K} \cdot \boldsymbol{R})$, with $\boldsymbol{R} = x\hat{\boldsymbol{x}} + y\hat{\boldsymbol{y}}$ the transversal position over the plane and $\boldsymbol{K} = k_x\hat{\boldsymbol{x}} + k_y\hat{\boldsymbol{y}}$ the spectral variable. Since the projection between two of the above basis functions, e.g., for two choices of $\boldsymbol{K}$, here $\boldsymbol{K}_1$ and $\boldsymbol{K}_2$, gives

$$\iint_{-\infty}^{\infty} \mathrm{d}\boldsymbol{R}\, f^*(\boldsymbol{R}; \boldsymbol{K}_1) f(\boldsymbol{R}; \boldsymbol{K}_2) = \delta(\boldsymbol{K}_1 - \boldsymbol{K}_2), \tag{2}$$

with $*$ the complex conjugate and $\delta(\cdot)$ Dirac's delta distribution, it is indeed possible to identify precisely the coefficient associated to each basis function. The identification is exact only in the case of data gathered over an infinitely large plane, i.e., the domain over which the orthogonality relationship (2) holds.

The projection, computed using the inner product, leads to defining the complex amplitude of the Fourier transform of $u(\boldsymbol{R}, z_o)$ as

$$\tilde{u}(\boldsymbol{K}, z_o) = \iint_{-\infty}^{\infty} \mathrm{d}\boldsymbol{R}\, u(\boldsymbol{R}, z_o) f^*(\boldsymbol{R}; \boldsymbol{K}) = \iint_{-\infty}^{\infty} \mathrm{d}\boldsymbol{R}\, u(\boldsymbol{R}, z_o) \mathrm{e}^{\mathrm{j}\boldsymbol{K}\cdot\boldsymbol{R}}, \tag{3}$$

and therefore to express $u(\boldsymbol{R}, z_o)$ as a (infinite) linear combination

$$u(\boldsymbol{R}, z_o) = \frac{1}{4\pi^2} \iint_{-\infty}^{\infty} \mathrm{d}\boldsymbol{K}\, \tilde{u}(\boldsymbol{K}, z_o) \mathrm{e}^{-\mathrm{j}\boldsymbol{K}\cdot\boldsymbol{R}}. \tag{4}$$

The above representation is known as PWE (or spectrum) or angular spectrum when expressed as a function of the director cosines of $\boldsymbol{K}$. Insofar it is essentially a mathematical procedure, with no physical-motivated rationale. The connection to plane wave propagation will be recalled in a moment.