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# Two-dimensional nonlinear stress and displacement waves for a new class of constitutive equations.

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#### Abstract

The propagation of displacement waves and stress waves for implicit constitutive equations is investigated. This new class of constitutive equations contain Cauchy elastic and hyperelastic bodies as subclasses. We consider a particular subclass where the strain is expressed in terms of a non-invertible function of the stress. This subclass of constitutive equations describe elastic responses where the stress and linearised strain are nonlinearly related. Such a phenomenon cannot be captured in the classical theory. Two constitutive equations are studied. The first constitutive equation is analogous to the constitutive equation for a powerlaw fluid with an exponent n in the expression for the stress and the second constitutive equation can describe elastic bodies which exhibit limiting strain. The special semi-inverse solution gives a system of nonlinear hyperbolic partial differential equations. These systems can be written as single partial differential equations which describe nonlinear shear stress waves. Solitary wave solutions for each constitutive equation are derived. Perturbation solutions for the system of partial differential equations for displacement and stress are considered and travelling wave and standing wave solutions are found. Although the solutions for both the travelling waves and standing waves contain a secular term, the perturbation expansions break down outside the range of interest. The speed of the solitary wave for both constitutive equations was obtained. The solitary waves develop a shock front and estimates for the time that this will occur are derived. The shock front develops at the back of the wave for the power-law constitutive equation and at the front of the wave for the strain-limiting constitutive equation. For the travelling waves the stress is non-zero at the wave front and the stress waves propagate as shock waves. The speed of propagation and the amplitude of the travelling waves and the period of oscillation of the standing waves are compared for the two constitutive equations.

*Keywords:* Implicit constitutive equations, nonlinear wave propagation, solitary waves, travelling waves, standing waves, shock wave.

#### 1. Introduction

The development of linear and nonlinear elasticity has centred on Cauchy elasticity and on Green elasticity also referred to as hyperelasticity. Cauchy elastic bodies are bodies where the stress is defined explicitly in terms of the deformation gradient and further, the stress is independent of the rate at which the material body deforms and the path taken during the deformation. In other words, the stress in the current configuration is determined by the state of deformation in the current configuration. Hyperelastic materials form a subclass of Cauchy elastic materials. These elastic bodies are bodies where the stress is derivable from a potential or stored energy associated with the body. The credence of Cauchy elastic bodies as being physically reasonable has been a topic of debate [1]. Rivilin [2] found that if the stress is not derived from a

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