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Second-harmonic generation in a multilayered structure with nonlinear spring-type interfaces embedded between two semi-infinite media

Yosuke Ishii [a,*], Shiro Biwa [b], Tadaharu Adachi [a]

[a] *Department of Mechanical Engineering, Toyohashi University of Technology, 1-1 Hibarigaoka, Tempaku-cho, Toyohashi 441-8580, Japan*
[b] *Department of Aeronautics and Astronautics, Graduate School of Engineering, Kyoto University, Katsura, Nishikyo-ku, Kyoto 615-8540, Japan*

HIGHLIGHTS

- The second-harmonic generation in a multilayered structure due to interfacial nonlinearity is analyzed.
- Explicit expressions for the second-harmonic amplitudes of the reflected and transmitted waves are derived using the transfer-matrix method.
- Influence of the position of nonlinear interfaces and the number of layers on the frequency dependence of the second-harmonic amplitudes is elucidated.

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ABSTRACT

The acoustic second-harmonic generation behavior in a multilayered structure with nonlinear spring-type interlayer interfaces is analyzed theoretically to investigate the frequency dependence of second-harmonic amplitudes in the reflected and transmitted fields when the structure is subjected to the normal incidence of a monochromatic longitudinal wave. The multilayered structure consists of identical linear elastic layers and is embedded between two identical linear elastic semi-infinite media. The layers are bonded to each other by spring-type interfaces possessing identical linear stiffness but different quadratic nonlinear parameters. By combining a perturbation analysis with the transfer-matrix method, analytical expressions are derived for the second-harmonic amplitudes of the reflected and transmitted waves. The second-harmonic amplitudes due to a single nonlinear interface are shown to vary remarkably with the fundamental frequency, reflecting the pass and stop band characteristics of the Bloch wave in the corresponding infinitely extended layered structure. By calculating the spatial distribution of second-harmonic amplitude inside the multilayered structure, the influence of the position of the nonlinear interface as well as the number of layers on the frequency dependence of second-harmonic amplitudes of the reflected and transmitted waves is elucidated. When all interlayer interfaces possess the identical nonlinearity, the second-harmonic amplitudes on both sides of the structure are shown to increase monotonically with the number of layers in the frequency ranges where both fundamental and double frequencies are within the pass bands of Bloch wave. The influence of two non-dimensional parameters, i.e., the relative linear compliance of

* Corresponding author.
E-mail address: ishii@me.tut.ac.jp (Y. Ishii).

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the interlayer interfaces and the acoustic impedance ratio between the layer and the surrounding semi-infinite medium, on the second-harmonic amplitudes is elucidated.

1. Introduction

Multilayered structures are found in various technological products and in nature such as advanced fiber-reinforced composite laminates in aerospace engineering, glued laminated timbers called glulam in architectural engineering, laminated rubber bearings for the seismic isolation in civil engineering, and stratified rocks in the Earth's crust. In such structures, many different types of imperfections can occur at the interfaces between neighboring layers: thin interphase layers, kissing bonds, closed cracks and delaminations, fractures, and so on. Understanding the influence of these imperfections on the wave propagation characteristics is essential from the viewpoints of the ultrasonic nondestructive testing in engineering practice, the seismic survey in oil and gas exploration, and the risk assessment of earthquake.

Foregoing studies have revealed that the elastic wave interaction with such imperfect interfaces can be analyzed by modeling them as spring-type interfaces [1–7]: the stresses are continuous while the displacements are allowed to be discontinuous across the interface, and the resulting jumps of displacements are related to the stresses by the proportional constants called interfacial stiffnesses. Spring-type interface models have been utilized extensively to characterize a contacting interface between rough surfaces of solid bodies [8–13], a partially closed crack [14,15], concentration of microcracks [16–18], a fracture in rock mass [19–23], adhesion at double interfaces between an adhesive layer and two adherents [24–28], multiple interlayer thin resin-rich zones of polymer-based composite laminates [29–33], and multiple rock joints [34,35].

On the other hand, when these imperfect interfaces are insonified by waves with sufficiently high amplitude, they can be a source of nonlinear acoustic phenomena. Among others, the higher-harmonic generation has been studied theoretically as well as experimentally in the field of ultrasonic nondestructive testing for more sensitive characterization of the interface quality than the conventional techniques based on the linear wave propagation characteristics [36–41]. According to the foregoing studies [42–48], the generation behavior of the second- or higher-order harmonics at a solid–solid contacting interface can be reasonably described by the nonlinear spring-type interface model. Using this model, Yan et al. [49,50] studied the second-harmonic generation at a kissing bond on one of the double interfaces between an adhesive layer and two aluminum blocks. The wave interaction with double nonlinear spring-type interfaces was also analyzed by Junca and Lombard [51]. For the more general case of multiple interfaces, Biwa and Ishii [52] analyzed the propagation characteristics of the Bloch wave in infinitely layered structures with spring-type interlayer interfaces possessing a weak quadratic nonlinearity. They elucidated the frequency dependence of the second-harmonic generation in the structures based on their pass and stop band characteristics. For finite layered structures with nonlinear spring-type interlayer interfaces, Ishii and Biwa [53] showed some preliminary numerical results of the frequency dependence of second-harmonic amplitudes of the reflected and transmitted waves by combining a perturbation approach with the transfer-matrix method [54,55], while they left out the details of the corresponding mathematical expressions. Better understanding of this issue is important from fundamental and practical points of view to characterize the imperfect interlayer interfaces of multilayered structures by using nonlinear acoustic methods.

In this paper, the formulations omitted in Ref. [53] is fully described and the second-harmonic generation behavior in multilayered structures is analyzed in a more precise manner. Namely, the present analysis deals with the second-harmonic generation in a finite layered structure with nonlinear spring-type interlayer interfaces when it is subjected to the normal incidence of a monochromatic longitudinal wave, and elucidate the frequency dependence of second-harmonic amplitudes in the reflected and transmitted fields. Following the perturbation analysis for a weak quadratic nonlinearity of interfaces carried out in Ref. [52], the governing equations for the propagation of fundamental wave and its second-harmonic component in the structure embedded between two linear elastic semi-infinite media are presented in the frequency domain in Section 2. The solution to the fundamental wave propagation, i.e., the amplitude reflection and transmission coefficients as well as the displacement jumps at the interlayer interfaces, is obtained using the transfer-matrix method in Section 3. The explicit expressions for the second-harmonic amplitudes of the reflected and transmitted waves are derived in Section 4. The resulting frequency dependence of the second-harmonic amplitude due to a single as well as multiple nonlinear interlayer interfaces is discussed in Section 5.

2. Formulation

The one-dimensional longitudinal wave propagation in the layering direction of a multilayered structure is considered. The position is denoted by x . The structure consists of N identical linear elastic layers (density ρ, wave velocity c, and thickness h) and is embedded between two identical linear elastic semi-infinite media (density ρ_0 and wave velocity c_0) with the perfect bonding at $x = X_0$ and $x = X_N$ as shown in Fig. 1. Using the displacement $u\,(x, t)$ where t is the time, the stress $\sigma\,(x, t)$ is given by

$$\sigma\,(x, t) = \rho c^2 \frac{\partial u}{\partial x}\,(x, t)\,, \tag{1}$$