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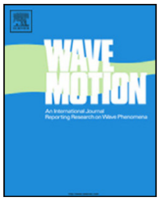

Study of acoustic wave propagation in micro- and nanochannels

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HIGHLIGHTS

- Second-order slip analytical model for acoustic propagation in micro/nano-channels.
- Numerical simulations using the molecular-based DSMC method to validate the model.
- Acoustics in porous nanomaterials such as aerogel, MEMS devices and metamaterials.

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ABSTRACT

The acoustic wave propagating through porous nanomaterials like aerogels, microelectromechanical systems (MEMS) devices, high-frequency acoustic transmission devices or near-vacuum systems, possesses relatively high Knudsen numbers, normally in the transition regime ($0.1 < \mathrm{Kn} < 10$). In this regime, the characteristic length of micro- and nanochannels is comparable with the mean free path of monatomic gases, in which the classical continuum theory breaks down. In this paper, a theoretical model with the second-order slip boundary is proposed to describe acoustic wave propagation in micro- and nanochannels. The proposed theoretical model provides analytical solutions for the complex wavenumber, attenuation coefficient and other related transmission variables as function of a Knudsen number in the early transition regime ($0.1 < \mathrm{Kn} < 1.0$), which are valuable for understanding acoustics at micro- and nanoscales. In addition, numerical simulations using the molecular-based direct simulation Monte Carlo (DSMC) method for dilute argon gas are carried out to validate the model and its analytical results. Findings suggest that such a model can effectively predict the acoustic behaviour in micro- and nanochannels.

1. Introduction

Advancing the understanding of wave propagation through rarefied gases in porous nanomaterials, MEMS-like inertial sensors, resonant filters, and actuators [1,2]-have been increasingly in demand. Continuum-based flow models have been often used to study the acoustic transmission issues analytically and numerically [3]. However, the extensive development of acoustical porous nanomaterials and miniaturization of MEMS devices at sub-atmospheric pressures, which cannot be modelled with sufficient accuracy by traditional continuum theory, demand a more fundamental understanding of acoustics at micro- and nanoscales. In contrast, an analytical theory or model is always preferable and valuable for practical uses.

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A theoretical model that is able to describe acoustic transmission in the transition regime can be applied to investigate sound propagation in micro- and nanostructures or in rarefied gas conditions, and therefore provide guidance in the design of nanomaterials and micro- or nanodevices. A detailed analysis of such phenomenon requires consideration of the rarefied effect, which is denoted by the Knudsen number as $Kn = \lambda/H$, where λ is the mean free path for a specific gas and H is the characteristic hydrodynamic length. Based on the Knudsen number, gas flow can be classified into three major flow regimes: the continuum regime ($\mathrm{Kn} < 0.1$), where the flow field can be described accurately by classical continuum theories; the transition regime ($0.1 < \mathrm{Kn} < 10$), where continuum-theory-based descriptions fail because the ratio of gas-structure collisions to intermolecular collisions becomes significant; and the free-molecular regime ($Kn > 10$), where there are nearly no collisions among the molecules [4].

Many MEMS devices and nanomaterials like aerogels fall in the early transition regime. The increasing popularity of nanomaterials and applications of MEMS devices demand more investigations of acoustics in the transition regime. Furthermore, these investigations can be applied to cases where either characteristic time or length scale is assumed to be smaller than that of the collision period or the mean free path of a specific gas, respectively. Such conditions can be found in the high-frequency acoustic transmission or near-vacuum systems [5]. Recently, realizations of negative effective density and slower effective acoustic speed of some acoustic metamaterials and lightweight sound absorbing materials like aerogel [6] also demand full investigation of acoustic transmission in the transition regime. In the area of topological acoustics [7], when the characteristic hydrodynamic length approaches the nanoscale, the related analytical theory of acoustics is also very important. Even for classical resonance structures, the analysis of acoustic mass, resistance and compliance may show some differences at the micro- and nanoscales. These potential applications indicate the importance of the study of acoustic wave propagation in the transition regime.

The pioneering theoretical and experimental research for acoustic wave propagation in the non-continuum regime was done by Greenspan [8] and Meyer et al. [9] in the 1950s, and has since attracted many researchers to study related issues. Initially, theoretical works were focused on acoustic propagation in infinite or semi-infinite geometries, where one moving boundary works as an acoustic source [5,10,11], and specific propagation properties like damping effects, wave speed and wavenumber in different external conditions (temperature, gases type and humidity) were studied. Afterwards, different geometrical cross-sections and gas-structure interaction models were proposed to investigate acoustic wave propagation in the transition regime analytically and numerically [12,13]. Later, contributions were made on a confined configuration, setting one stationary boundary as a "resting receiver" to study the effects of acoustic wave reflection and other related transmission issues [14–16]. The majority of studies in this area have assumed linearized small-amplitude acoustic wave solutions, while few have examined nonlinear large-amplitude issues [17].

To investigate wave propagation in small confined geometries with specific boundary conditions, Hadjiconstantinou [18] in 2003 developed a model with the first-order slip boundary condition and provided an analytical solution for acoustic wave propagation in micro- and nanochannels, which was an extension of Lamb's continuum treatment [19,20]. In 2005, considering the effects of the Knudsen layer and using the second-order velocity boundary condition, Hadjiconstantinou [21] solved the oscillatory shear-driven Couette flow problem using the Navier–Stokes approximation and achieved good agreement with DSMC results up to $\mathrm{Kn} \approx 0.4$. Kozlov et al. [12] investigated acoustical properties in pores of simple geometries with the first-order approximation and validated their results with experimental data of dynamic density at low frequencies. Umnova et al. [13] developed an analytical model to describe acoustic propagation in microfibrous materials while accounting for the slip boundary effect, where the homogeneous method used was also verified by finite element method (FEM) simulation. In terms of numerical investigation, various simulation methods have been proposed, such as the linearized Boltzmann method [22–24], Lattice Boltzmann method [25–27], Bhatnagar–Gross–Krook (BGK) model [28,29], molecular dynamics (MD) model [30–32], and DSMC [33,34]. Based on our literature survey, the DSMC method, initially proposed by G. A. Bird [33], is the most widely used tool for the simulation of acoustic wave propagation at the micro- and nanoscales and is renowned for its accuracy and time efficiency. Derivative DSMC methods, such as Wang and Xu's unified gas-kinetic scheme DSMC (UGKS-DSMC) [35], Fan and Shen's information preserving DSMC (IP-DSMC) [36], Mohssen and Hadjiconstantinou's Low-Variance Deviational Simulation Monte Carlo (LVDSMC) [37], were proposed to improve the performance of the traditional DSMC method. All these works mentioned above can only extend the traditional continuum theory up to $\mathrm{Kn} \approx 0.4$ or even smaller for the acoustic wave propagation in micro- and nanochannels, so there is still a need to develop an analytical solution suitable for higher Knudsen numbers.

In this paper, we investigate acoustic wave propagation in micro- and nanochannels that fall in the transition regime. Specifically, an accurate and easy-to-use theoretical model and associated analytical solutions is developed using the second-order velocity and temperature slip boundary conditions, and verified by DSMC simulations. Based on the modified acoustic slip surface definition of the second-order slip boundary condition in gas flow, we develop a theoretical and analytical model that is able to provide a detailed description of acoustic wave propagation at relatively high Kn numbers, up to 1.0. Based on this model, we can extract properties such as the acoustic propagation constant, complex effective density, effective acoustic speed and characteristic impedance. A DSMC program based on Bird's DS2V simulation tool is developed to validate our analytical results in the transition regime.

2. Theory for acoustic wave propagation in narrow channels

The basic wave propagation theory in narrow channels is derived from Hadjiconstantinou's method [18], and some data setups are based on Hadjiconstantinou and Garcia's paper [38]. Specifically, the long-wavelength approximation is used to