Wave Motion 68 (2017) 22–30

Contents lists available at ScienceDirect

Wave Motion

journal homepage: www.elsevier.com/locate/wavemoti

Shear to longitudinal mode conversion via second harmonic generation in a two-dimensional microscale granular crystal

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HIGHLIGHTS

- Second harmonic generation in a 2D microscale granular crystal model is studied.
- The model includes particle rotations and nonlinear normal and shear contact laws.
- Second harmonics are longitudinal for transverse and longitudinal fundamental modes.
- Resonant and antiresonant wave numbers are found for transverse fundamental modes.
- The antiresonance is prohibited if rotations are not included in the model.

ARTICLE INFO

Article history:
Received 3 February 2016
Received in revised form 12 August 2016
Accepted 16 August 2016
Available online 25 August 2016

Keywords:
Granular media
Nonlinear dynamics
Periodic structures

ABSTRACT

Shear to longitudinal mode conversion via second harmonic generation is studied theoretically and computationally for plane waves in a two-dimensional, adhesive, hexagonally close-packed microscale granular medium. The model includes translational and rotational degrees of freedom, as well as normal and shear contact interactions. We consider fundamental frequency plane waves in all three linear modes, which have infinite spatial extent and travel in one of the high-symmetry crystal directions. The generated second harmonic waves are longitudinal for all cases. For the lower transverse–rotational mode, an analytical expression for the second harmonic amplitude, which is derived using a successive approximations approach, reveals the presence of particular resonant and antiresonant wave numbers, the latter of which is prohibited if rotations are not included in the model. By simulating a lattice with adhesive contact force laws, we study the effectiveness of the theoretical analysis for non-resonant, resonant, and antiresonant cases. This work is suitable for the analysis of microscale and statically compressed macroscale granular media, and should inspire future studies on nonlinear two- and three-dimensional granular systems in which interparticle shear coupling and particle rotations play a significant role.

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1. Introduction

Granular media are known to exhibit complex dynamical behavior that stems from their discrete, often heterogeneous, structure and highly nonlinear particulate interactions [1–3]. Ordered and reduced-dimensional granular systems, often referred to as "granular crystals," have been a setting of interest for several decades, as they have yielded new, broader understanding of granular media dynamics [3]. In addition, utilization of the nonlinear particulate interactions in conjunction with classical linear effects such as dispersion induced by structural periodicity has resulted in granular crystals being used in new strategies for passive wave tailoring [4,5].

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http://dx.doi.org/10.1016/j.wavemoti.2016.08.009
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Models used to describe granular crystals are typically composed of one- to three-dimensional (1D and 3D, respectively) systems of rigid bodies interacting via Hertzian normal contact "springs" [6,7]. For example, this approach has been used in previous studies to explore highly nonlinear mechanical wave propagation in uncompressed two-dimensional (2D) granular crystals [3,8–12]. While this type of model works well for uncompressed granular media, interparticle shear interactions and particle rotations become increasingly important in statically compressed granular systems. Several recent theoretical works, based on earlier discrete lattice models of elastic media [13], have introduced linear models of 2- [14,15] and 3D [16] granular crystals that account for interparticle shear interactions and particle rotations. These studies demonstrated how the additional degrees of freedom and modes of particle coupling can drastically influence the granular crystal dynamics, and yield unique effects such as the rotational waves experimentally observed in statically compressed macroscale granular systems [17]. Shear interactions and particle rotations also play a similarly important role in the emerging field of microscale granular crystals, as has been demonstrated in recent theoretical [18] and experimental [19] studies of quasi-1D microgranular systems in linear dynamical regimes.

In this work, we explore the nonlinear phenomenon of second harmonic generation for plane waves traveling in a model of a 2D hexagonally close-packed lattice of microspheres, which includes interparticle adhesive effects, particle rotations, and elastic shear interactions. Second harmonic generation is a well-known phenomenon that has been studied extensively in nonlinear optics [20] and acoustics [21]. Past works have also examined second harmonic generation in 1D discrete granular chains [22,23] and fluid-saturated granular media [24]. Using a successive approximations approach in the manner of Refs. [22,23], we theoretically analyze the second harmonic generation for cases where the fundamental frequency (FF) wave is purely longitudinal or transverse–rotational in character, and find that the generated second harmonic waves are longitudinal in both scenarios. Such shear-to-longitudinal mode conversion via second harmonic generation has been previously studied in nonlinear elastic solids [25], observed experimentally in 3D granular packings [26,27], and modeled in quasi-1D nonlinear phononic crystals [28]. We show that the second harmonic generated by a FF wave in the lower transverse–rotational mode may resonate or vanish for particular wavelengths, but the latter phenomenon is not predicted if rotations are excluded from the model. Finally, we compare our theoretical predictions with dynamic discrete element simulations of a microscale granular crystal. We find that the theoretical predictions are quantitatively accurate for non-resonant wavelengths, and provide qualitative understanding of the behavior at resonance. This paper extends the already rich body of work on nonlinear waves in 2- and 3D ordered granular media by exploring the interplay of multiple degrees of freedom and nonlinear particulate interactions.

2. Theory

2.1. Model

We consider a 2D, hexagonally close-packed lattice of spheres, as shown in Fig. 1(a). The interactions between spheres follow the Derjaguin–Muller–Toporov (DMT) adhesion model [29,30], which includes Hertzian normal contact forces [6,7], and a static adhesive force $F_{DMT} = 2\pi wR_c$ due to van der Waals interactions, where w is the work of adhesion [30], $R_c = R/2$ is the effective radius for two spheres in contact, and R is the microsphere radius. To describe the shear contact interactions, we use the Hertz–Mindlin model [7,31], assuming no slip occurs in the contact surface. Because the characteristic sound speeds of waves in the lattice are much slower than those of the bulk material of the spheres, we treat the spheres as rigid bodies with radius R, mass m, and moment of inertia $I = (2/5)mR^2$, interacting via nonlinear spring elements [3]. In terms of the displacements $u_{j,k}$, $v_{j,k}$, and $\theta_{j,k}$, which represent horizontal, vertical, and angular displacements from equilibrium, respectively, the equations of motion of the sphere with index (j, k) are given by

$$m\ddot{u}_{j,k} = f_N(\bar{\delta}_2) - f_N(\bar{\delta}_5) + \frac{1}{2}(f_N(\bar{\delta}_1) - f_N(\bar{\delta}_4) + f_N(\bar{\delta}_3) - f_N(\bar{\delta}_6)) + \sqrt{\frac{3}{2}}(f_S(\bar{\delta}_3) - f_S(\bar{\delta}_6) - f_S(\bar{\delta}_1) + f_S(\bar{\delta}_4)) \tag{1}$$

$$m\ddot{v}_{j,k} = f_S(\bar{\delta}_2) - f_S(\bar{\delta}_5) + \frac{1}{2}(f_S(\bar{\delta}_1) - f_S(\bar{\delta}_4) + f_S(\bar{\delta}_3) - f_S(\bar{\delta}_6)) + \sqrt{\frac{3}{2}}(f_N(\bar{\delta}_1) - f_N(\bar{\delta}_4) - f_N(\bar{\delta}_3) + f_N(\bar{\delta}_6)) \tag{2}$$

$$I\ddot{\theta}_{j,k} = R\sum_{l} f_S(\bar{\delta}_l), \tag{3}$$

where $\bar{\delta}_l = (\delta_{l,N}, \delta_{l,S})$ is the vector of relative normal and tangential displacements between particles labeled l and 0, as is defined in the Appendix. The normal spring force (positive away from particle 0) and shear spring force (positive when inducing a counter-clockwise moment about particle 0) are given by

$$f_N(\bar{\delta}_l) = -\frac{4}{3}E^*R_c^{1/2}\left[\Delta_0 - \delta_{l,N}\right]_+^{3/2} \tag{4}$$

$$f_S(\bar{\delta}_l) = 8G^*R_c^{1/2}\delta_{l,S}\left[\Delta_0 - \delta_{l,N}\right]_+^{1/2}, \tag{5}$$

respectively. Here, $\Delta_0 = [3F_{DMT}/(4E^*R_c^{1/2})]^{2/3}$ is the static overlap due to adhesion, and $E^* = E/(2(1-\nu^2))$ and $G^* = G/(2(2-\nu))$ are the effective elastic and shear moduli, respectively, of a solid with elastic modulus E, shear modulus

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